CHAPTER 15

Electromagnetic Induction and Alternating Current

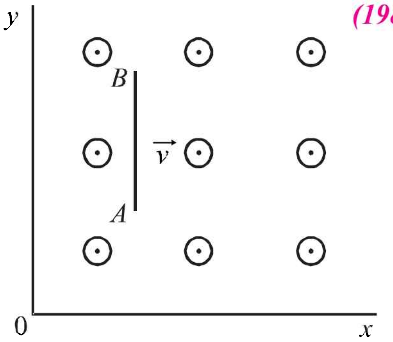
Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. A uniformly wound solenoidal coil of self inductance 1.8×10^{-4} henry and resistance 6 ohm is broken up into two identical coils. These identical coils are then connected in parallel across a 15-volt battery of negligible resistance. The time constant for the current in the circuit is .... seconds and the steady state current through the battery is .... amperes. *(1989 - 2 Marks)*
2. In a straight conducting wire, a constant current is flowing from left to right due to a source of emf. When the source is switched off, the direction of the induced current in the wire will .... *(1993 - 1 Marks)*

B True/False

1. An e.m.f. can be induced between the two ends of a straight copper wire when it is moved through a uniform magnetic field. *(1980)*
2. A coil of metal wire is kept stationary in a non-uniform magnetic field. An e.m.f. is induced in the coil. *(1986 - 3 Marks)*
3. A conducting rod AB moves parallel to the x-axis (see Fig.) in a uniform magnetic field pointing in the positive z-direction. The end A of the rod gets positively charged. *(1987 - 2 Marks)*

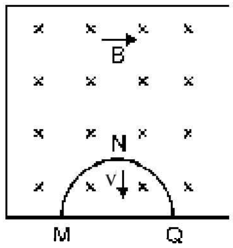


C MCQs with One Correct Answer

1. A thin circular ring of area A is held perpendicular to a uniform magnetic field of induction B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is *(1995S)*

   (a) $\frac{BR}{A}$ (b) $\frac{AB}{R}$ (c) ABR (d) $\frac{B^2A}{R^2}$

2. A thin semi-circular conducting ring of radius R is falling with its plane vertical in horizontal magnetic induction $\vec{B}$. At the position MNQ the speed of the ring is v, and the potential difference developed across the ring is



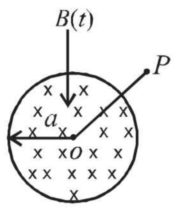
   (a) zero
   (b) $Bv\pi R^2/2$ and M is at higher potential
   (c) πRBv and Q is at higher potential *(1996 - 2 Marks)*
   (d) $2RBv$ and Q is at higher potential.

3. Two identical circular loops of metal wire are lying on a table without touching each other. Loop-A carries a current which increases with time. In response, the loop-B
   (a) remains stationary *(1999S - 2 Marks)*
   (b) is attracted by the loop-A
   (c) is repelled by the loop-A
   (d) rotates about its CM, with CM fixed

4. A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12 V battery. The current in the coil is 1.0 A at approximately the time *(1999S - 2 Marks)*

   (a) 500 s (b) 25 s (c) 35 ms (d) 1 ms

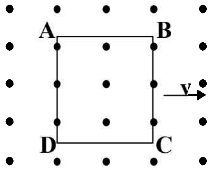
5. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region *(2000S)*



   (a) is zero
   (b) decreases as $1/r$
   (c) increases as r
   (d) decreases as $1/r^2$

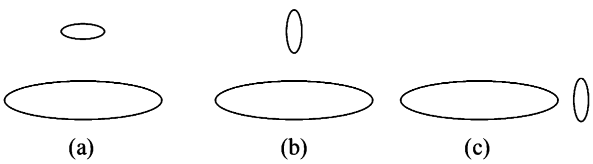
6. A coil of wire having inductance and resistance has a conducting ring placed coaxially within it. The coil is connected to a battery at time $t = 0$, so that a time-dependent current $I_1(t)$ starts flowing through the coil. If $I_2(t)$ is the current induced in the ring, and $B(t)$ is the magnetic field at the axis of the coil due to $I_1(t)$, then as a function of time ($t > 0$), the product $I_2(t)\,B(t)$ *(2000S)*
   (a) increases with time
   (b) decreases with time
   (c) does not vary with time
   (d) passes through a maximum

7. A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. An electric field is induced *(2001S)*



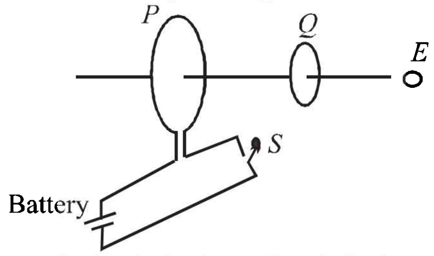
(a) in AD, but not in BC (b) in BC, but not in AD
(c) neither in AD nor in BC (d) in both AD and BC

8. Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be *(2001S)*



(a) maximum in situation (a) (b) maximum in situation (b)
(c) maximum in situation (c) (d) the same in all situations

9. As shown in the figure, P and Q are two coaxial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q2} flows in Q. Then the direction IQ_1 and IQ_2 (as seen by E) are *(2002S)*

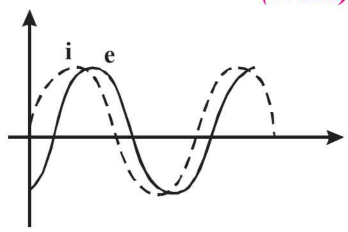


(a) respectively clockwise and anti-clockwise
(b) both clockwise
(c) both anti-clockwise
(d) respectively anti-clockwise and clockwise

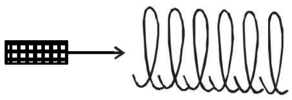
10. A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be *(2002S)*
(a) halved (b) the same
(c) doubled (d) quadrupled

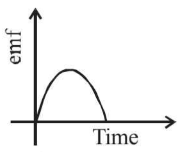
11. When an AC source of emf $e = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the emf e and the current i in the circuit is observed to be $\pi/4$, as shown in the diagram. If the circuit consists possibly only of R-C or R-L or L-C in series, find the relationship between the two elements *(2003S)*

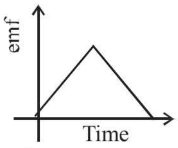
(a) $R = 1k\Omega, C = 10\ \mu F$
(b) $R = 1k\Omega, C = 1\ \mu F$
(c) $R = 1k\Omega, L = 10\ H$
(d) $R = 1k\Omega, L = 1\ H$



12. A small bar magnet is being slowly inserted with constant velocity inside a solenoid as shown in figure. Which graph best represents the relationship between emf induced with time *(2004S)*



(a)


(b)


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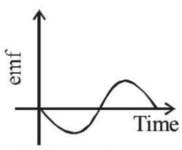
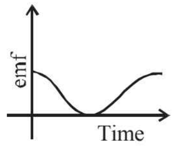
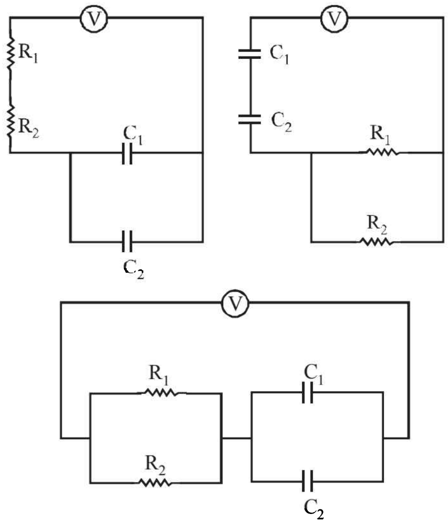


(d)


13. An infinitely long cylinder is kept parallel to an uniform magnetic field B directed along positive z-axis. The direction of induced current as seen from the z-axis will be *(2005S)*
(a) zero
(b) anticlockwise of the +ve z axis
(c) clockwise of the +ve z axis
(d) along the magnetic field

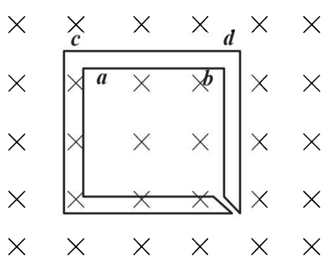
14. Find the time constant (in μs) for the given RC circuits in the given order respectively *(2006 - 3M, –1)*



$R_1 = 1\Omega,\ R_2 = 2\Omega,\ C_1 = 4\mu F,\ C_2 = 2\mu F$

(a) $18, 4, \frac{8}{9}$ (b) $18, \frac{8}{9}, 4$ (c) $4, 18, \frac{8}{9}$ (d) $4, \frac{8}{9}, 18$

15. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments ***ab*** and ***cd***. Then, *(2009)*



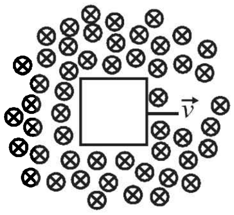
(a) $I_1 > I_2$
(b) $I_1 < I_2$
(c) I_1 is in the direction ***ba*** and I_2 is in the direction ***cd***
(d) I_1 is in the direction ***ab*** and I_2 is in the direction ***dc***

16. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased *(2010)*
(a) the bulb glows dimmer
(b) the bulb glows brighter
(c) total impedance of the circuit is unchanged
(d) total impedance of the circuit increases

D MCQs with One or More than One Correct

1. L, C and R represent the physical quantities, inductance, capacitance and resistance respectively. The combination(s) which have the dimensions of frequency are *(1984- 2 Marks)*
(a) $1/RC$ (b) R/L (c) $1/\sqrt{LC}$ (d) C/L

2. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular and into the plane of the loop exists everywhere. *(1989 - 2 Marks)*



The current induced in the loop is:
(a) BLv/R clockwise (b) BLv/R anticlockwise
(c) $2BLv/R$ anticlockwise (d) zero.

3. Two different coils have self-inductances $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then: *(1994 - 2 Marks)*
(a) $\frac{i_1}{i_2} = \frac{1}{4}$ (b) $\frac{i_1}{i_2} = 4$ (c) $\frac{W_1}{W_2} = \frac{1}{4}$ (d) $\frac{V_1}{V_2} = 4$

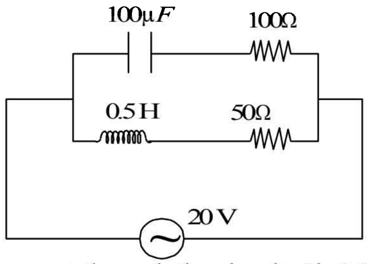
4. A small square loop of wire of side l is placed inside a large square loop of wire of side $L(L>>l)$. The loops are co-planar and their centres coincide. The mutual inductance of the system is proportional to *(1998S - 2 Marks)*
(a) l/L (b) l^2/L (c) L/l (d) L^2/l

5. The SI unit of inductance, the henry, can be written as *(1998S - 2 Marks)*
(a) weber/ampere (b) volt-second/ampere
(c) joule/(ampere)^2 (d) ohm-second

6. A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant, uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement(s) from the following *(1998S - 2 Marks)*
(a) The entire rod is at the same electric potential.
(b) There is an electric field in the rod.
(c) The electric potential is highest at the centre of the rod and decreases towards its ends.
(d) The electric potential is lowest at the centre of the rod, and increases towards its ends

7. A series R– C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_c across the capacitor are compared in the two cases. Which of the following is/are true? *(2011)*
(a) $I_R^A > I_R^B$ (b) $I_R^A < I_R^B$
(c) $V_C^A > V_C^B$ (d) $V_C^A < V_C^B$

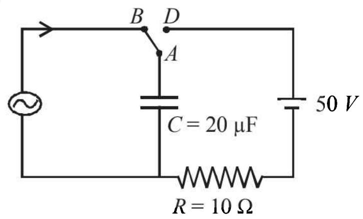
8. In the given circuit, the AC source has ω = 100 rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is (are) *(2012)*



(a) The current through the circuit, I is 0.3 A.
(b) The current through the circuit, I is $0.3\sqrt{2}A$
(c) The voltage across 100 Ω resistor = $10\sqrt{2}V$
(d) The voltage across 50 Ω resistor = 10 V

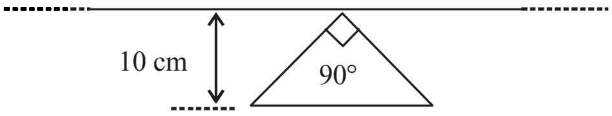
9. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it, the correct statement(s) is(are) *(2012)*
(a) The emf induced in the loop is zero if the current is constant.
(b) The emf induced in the loop is finite if the current is constant.
(c) The emf induced in the loop is zero if the current decreases at a steady rate.
(d) The emf induced in the loop is infinite if the current decreases at a steady rate.

10. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1$ A and $\omega = 500$ rad s^{-1} starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20$ μF, R = 10 Ω and the battery is ideal with emf of 50 V, identify the correct statement(s). *(JEE Adv. 2014)*



(a) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3} C
(b) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise
(c) Immediately after A is connected to D, the current in R is 10 A
(d) $Q = 2 \times 10^{-3}$ C

11. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 A s^{-1}. Which of the following statement(s) is(are) true? ***(JEE Adv. 2016)***



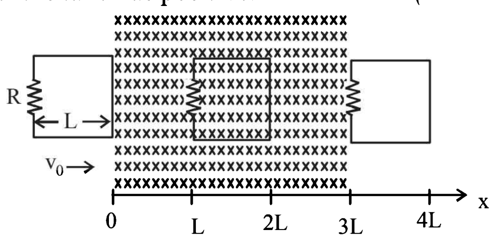
(a) The magnitude of induced *emf* in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

(b) If the loop is rotated at a constant angular speed about the wire, an additional *emf* of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

(c) The induced current in the wire is in opposite direction to the current along the hypotenuse

(d) There is a repulsive force between the wire and the loop

12. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At t = 0, the right edge of the loop enters a region of length 3L where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let v(x), I(x) and F(x) represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive. ***(JEE Adv. 2016)***



Which of the following schematic plot(s) is (are) correct? (Ignore gravity)

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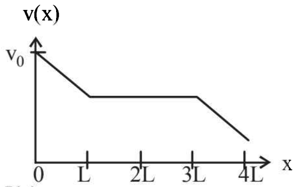
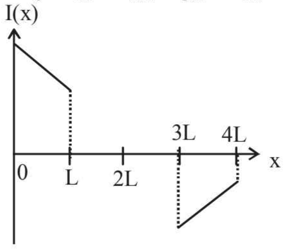
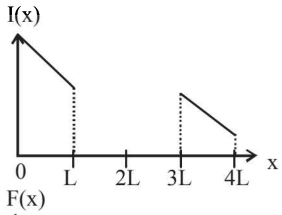
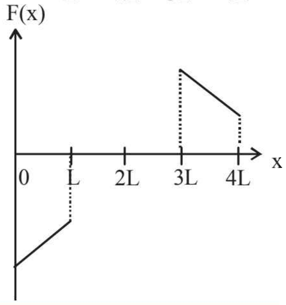


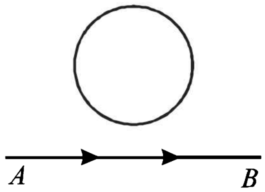
(b)


(c)


(d)


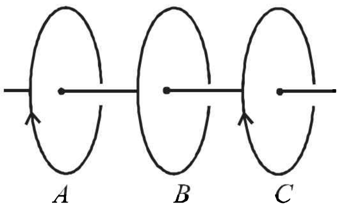
E Subjective Problems

1. A current from *A* to *B* is increasing in magnitude. What is the direction of induced current, if any, in the loop as shown in the figure? ***(1979)***

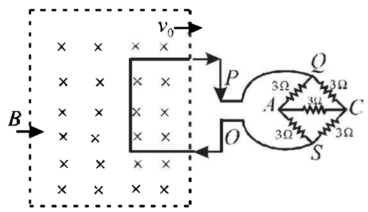


2. The two rails of a railway track, insulated from each other and the ground, are connected to a milli voltmeter. What is the reading of the milli voltmeter when a train travels at a speed of 180 km/hour along the track, given that the vertical component of earth's magnetic field is 0.2×10^{-4} weber/m^2 & the rails are separated by 1 meter? ***(1981- 4 Marks)***

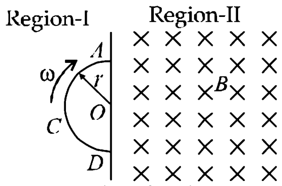
3. Three identical closed coils *A*, *B* and *C* are placed with their planes parallel to one another. Coils *A* and *C* carry equal currents as shown in Fig. Coils *B* and *C* are fixed in position and coil *A* is moved towards *B* with uniform motion. Is there any induced current in *B* ? If no, give reasons. If yes mark the direction of the induced current in the diagram. ***(1982 - 2 Marks)***



4. A square metal wire loop of side 10 cms and resistance 1 ohm is moved with a constant velocity v_0 in a uniform magnetic field of induction *B* = 2 webers/m^2 as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to a network of resistors each of value 3 ohms. The resistances of the lead wires *OS* and *PQ* are negligible. What should be the speed of the loop so as to have a steady current of 1 milliampere in the loop ? Give the direction of current in the loop. ***(1983 - 6 Marks)***

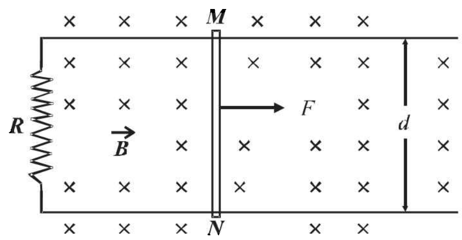


5. Space is divided by the line *AD* into two regions. Region I is field free and the Region II has a uniform magnetic field *B* directed into the plane of the paper. *ACD* is a semicircular conducting loop of radius *r* with centre at *O*, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through *O* and the perpendicular to the plane of the paper. The effective resistance of the loop is *R*. ***(1985 - 6 Marks)***



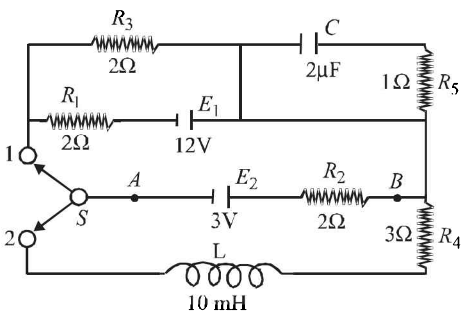
(i) Obtain an expression for the magnitude of the induced current in the loop.
(ii) Show the direction of the current when the loop is entering into the Region II.
(iii) Plot a graph between the induced e.m.f and the time of rotation for two periods of rotation.

6. Two long parallel horizontal rails, a distance *d* apart and each having a resistance λ per unit length, are joined at one end by a resistance *R*. A perfectly conducting rod *MN* of mass m is free to slide along the rails without friction (see figure). There is a uniform magnetic field of induction *B* normal to the plane of the paper and directed into the paper. A variable force *F* is applied to the rod *MN* such that, as the rod moves, a constant current flows through *R*. ***(1988 - 6 Marks)***



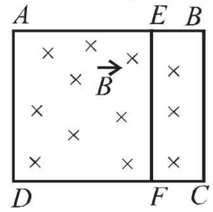
(i) Find the velocity of the rod and the applied force *F* as function of the distance *x* of the rod from *R*.
(ii) What fraction of the work done per second by *F* is converted into heat ?

7. A circuit containing a two position switch S is shown in fig. ***(1991 - 4 + 4 Marks)***

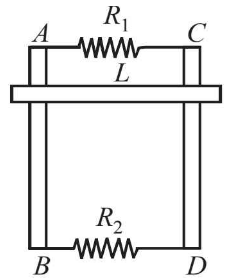


(a) The switch *S* is in position '1'. Find the potential difference $V_A - V_B$ and the rate of production of joule heat in R_1.
(b) If now the switch *S* is put in position 2 at $t = 0$ find
(i) steady current in R_4 and
(ii) the time when current in R_4 is half the steady value.
Also calculate the energy stored in the inductor *L* at that time

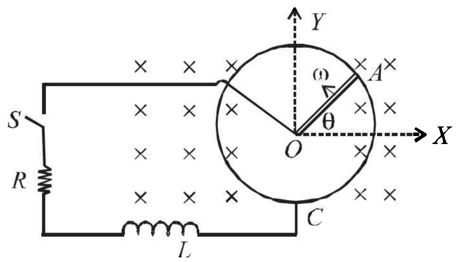
8. A rectangular frame *ABCD*, made of a uniform metal wire, has a straight connection between *E* and *F* made of the same wire, as shown in Fig. *AEFD* is a square of side 1m, and $EB = FC = 0.5$m. The entire circuit is placed in steadily increasing, uniform magnetic field directed into the plane of the paper and normal to it. The rate of change of the magnetic field is 1T/s. The resistance per unit length of the wire is $1\Omega/\text{m}$. Find the magnitudes and directions of the currents in the segments *AE, BE* and *EF*. ***(1993-5 Marks)***



9. Two parallel vertical metallic rails *AB* and *CD* are separated by 1 m. They are connected at two ends by resistances R_1 and R_2 as shown in Figure. A horizontal metallic bar *L* of mass 0.2 kg slides without friction vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 Tesla perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in R_1 and R_2 are 0.76 Watt and 1.2 watt respectively. Find the terminal velocity of the bar *L* and the values of R_1 and R_2. ***(1994 - 6 Marks)***



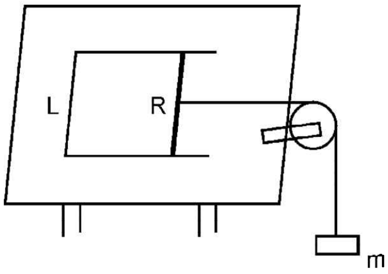
10. A metal rod *OA* of mass '*m*' and length '*r*' is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end *O*. The free end *A* is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction $\vec{B}$ is applied perpendicular and into the plane of rotation as shown in the figure below. An inductor L and an external resistance *R* are connected through a switch *S* between the point *O* and a point *C* on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open. ***(1995 - 10 Marks)***



(a) What is the induced emf across the terminals of the switch?
(b) The switch *S* is closed at time $t = 0$.
(i) Obtain an expression for the current as a function of time.
(ii) In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed, given that the rod *OA* was along the positive X-axis at $t = 0$.

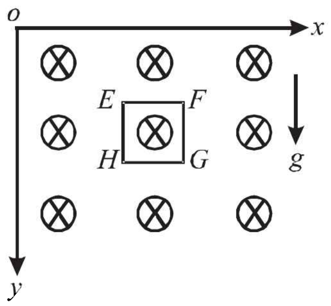
11. A solenoid has an inductance of 10 henry and a resistance of 2 ohm. It is connected to a 10 volt battery. How long will it take for the magnetic energy to reach 1/4 of its maximum value? ***(1996 - 3 Marks)***

12. A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails frictionlessly. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m, tied to the other end of the string hangs vertically. A constant magnetic field B exists perpendicular to the table. If the system is released from rest, calculate. *(1997 - 5 Marks)*



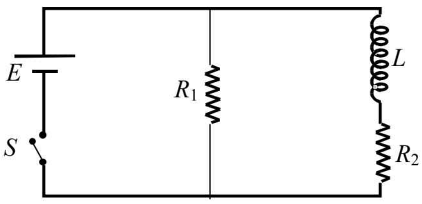
(i) the terminal velocity achieved by the rod, a nd
(ii) the acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity.

13. A magnetic field $B = B_0\,(y/a)\hat{k}$ is into the paper in the $+z$ direction. B_0 and a are positive constants. A square loop $EFGH$ of side a, mass m and resistance R, in $x - y$ plane, starts falling under the influence of gravity see figure) Note the directions of x and y axes in figure. *(1999 - 10 Marks)*)

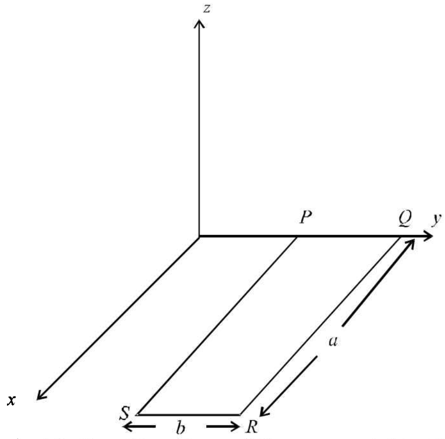


Find
(a) the induced current in the loop and indicate its direction.
(b) the total Lorentz force acting on the loop and indicate its direction, and
(c) an expression for the speed of the loop, $v(t)$ and its terminal value.

14. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\Omega$ and $R_2 = 2\ \Omega$ are connected to a battery of emf $E = 12$ V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at time $t = 0$. What is the potential drop across L as a function of time? After the steady state is reached, the switch is opened. What is the direction and the magnitude of current through R_1 as a function of time? *(2001-5 Marks)*

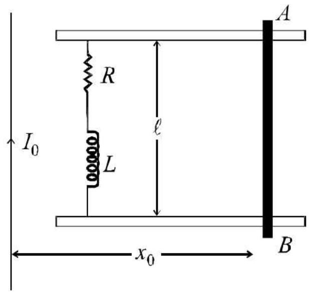


15. A rectangular loop $PQRS$ made from a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis (see figure). Take the vertically upward direction as the z-axis. A uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{k})B_0$ exists in the region. The loop is held in the x-y plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium. *(2002 - 5 Marks)*



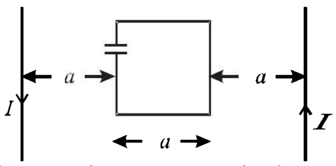
(a) What is the direction of the current I in PQ?
(b) Find the magnetic force on the arm RS.
(c) Find the expression for I in terms of B_0, a, b and m.

16. A metal bar AB can slide on two parallel thick metallic rails separated by a distance ℓ. A resistance R and an inductance L are connected to the rails as shown in the figure. A long straight wire carrying a constant current I_0 is placed in the plane of the rails and perpendicular to them as shown. The bar AB is held at rest at a distance x_0 from the long wire. At $t = 0$, it is made to slide on the rails away from the wire. Answer the following questions. *(2002 - 5 Marks)*



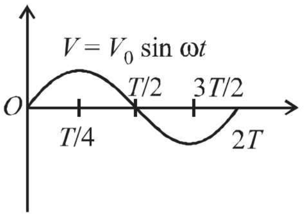
(a) Find a relation among i, $\frac{di}{dt}$ and $\frac{d\phi}{dt}$, where i is the current in the circuit and ϕ is the flux of the magnetic field due to the long wire through the circuit.
(b) It is observed that at time $t = T$, the metal bar AB is at a distance of $2x_0$ from the long wire and the resistance R carries a current i_1. Obtain an expression for the net charge that has flown through resistance R from $t = 0$ to $t = T$.
(c) The bar is suddenly stopped at time T. The current through resistance R is found to be $\frac{i_1}{4}$ at time $2T$. Find the value of $\frac{L}{R}$ in terms of the other given quantities.

17. A square loop of side 'a' with a capacitor of capacitance C is located between two current carrying long parallel wires as shown. The value of I in the wires is given as $I = I_0 \sin\omega t$. *(2003 - 4 Marks)*

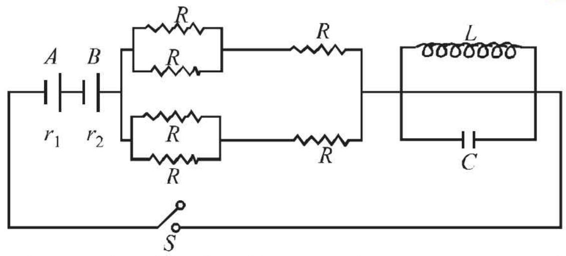


(a) Calculate maximum current in the square loop.
(b) Draw a graph between charges on the upper plate of the capacitor vs time.

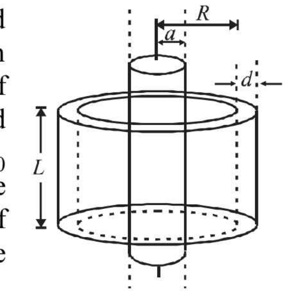
18. In a series L–R circuit ($L = 35$ mH and $R = 11\ \Omega$), a variable emf source ($V = V_0 \sin \omega t$) of $V_{rms} = 220$ V and frequency 50 Hz is applied. Find the current amplitude in the circuit and phase of current with respect to voltage. Draw current-time graph on given graph ($\pi = 22/7$). ***(2004 - 4 Marks)***



19. In the figure both cells A and B are of equal emf. Find R for which potential difference across battery A will be zero, long time after the switch is closed. Internal resistance of batteries A and B are r_1 and r_2 respectively ($r_1 > r_2$). ***(2004 - 4 Marks)***



20. A long solenoid of radius a and number of turns per unit length n is enclosed by cylindrical shell of radius R. thickness d ($d << R$) and length L. A variable current $i = i_0 \sin\omega t$ flows through the coil. If the resistivity of the material of cylindrical shell is ρ, find the induced current in the shell.

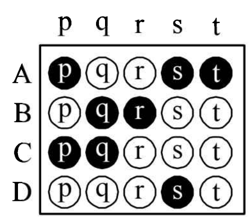


(2005 - 4 Marks)

F Match the Following

DIRECTIONS : *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.

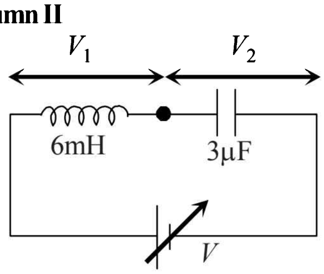


1. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in **Column II**. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2, (indicated in circuits) are related as shown in **Column I.** Match the two ***(2010)***

Column I **Column II**

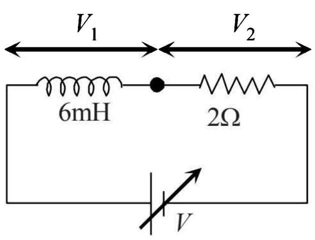
(A) $I \neq 0, V_1$ is proportional to I

(p)



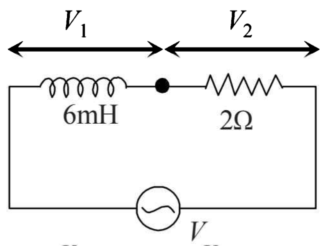
(B) $I \neq 0, V_2 > V_1$

(q)



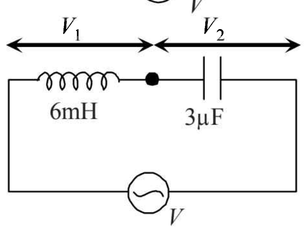
(C) $V_1 = 0, V_2 = V$

(r)

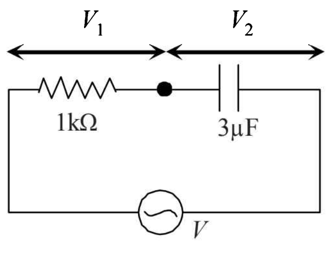


(D) $I \neq 0, V_2$ is proportional to I

(s)



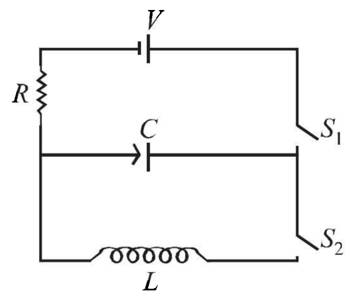
(t)



G Comprehension Based Questions

PASSAGE 1

In the given circuit the capacitor (C) may be charged through resistance R by a battery V by closing switch S_1. Also when S_1 is opened and S_2 is closed the capacitor is connected in series with inductor (L).



1. At the start, the capacitor was uncharged. When switch S_1 is closed and S_2 is kept open, the time constant of this circuit is τ. Which of the following is correct *(2006 – 5M, –2)*
   (a) after time interval τ, charge on the capacitor is $\frac{CV}{2}$
   (b) after time interval 2τ, charge on the capacitor of $CV(1-e^{-2})$
   (c) the work done by the voltage source will be half of the heat dissipated when the capacitor is fully charged
   (d) after time interval 2τ, charge on the capacitor is $CV(1-e^{-1})$
2. When the capacitor gets charged completely, S_1 is opened and S_2 is closed. Then, *(2006 – 5M, –2)*
   (a) at $t = 0$, energy stored in the circuit is purely in the form of magnetic energy
   (b) at any time $t > 0$, current in the circuit is in the same direction
   (c) at $t > 0$, there is no exchange of energy between the inductor and capacitor
   (d) at any time $t > 0$, instantaneous current in the circuit may be $V\sqrt{\frac{C}{L}}$
3. Given that the total charge stored in the LC circuit is Q_0, for $t \geq 0$, the charge on the capacitor is *(2006 – 5M, –2)*
   (a) $Q = Q_0 \cos\left(\frac{\pi}{2}+\frac{t}{\sqrt{LC}}\right)$ (b) $Q = Q_0 \cos\left(\frac{\pi}{2}-\frac{t}{\sqrt{LC}}\right)$
   (c) $Q = -LC\frac{d^2Q}{dt^2}$ (d) $Q = -\frac{1}{\sqrt{LC}}\frac{d^2Q}{dt^2}$

PASSAGE 2

A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method , a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with power factor unity. All the currents and voltages mentioned are rms values. *(JEE Adv. 2013)*

4. If the direct transmission method with a cable of resistance 0.4 Ω km^{-1} is used, the power dissipation| (in %) during transmission is
   (a) 20 (b) 30 (c) 40 (d) 50
5. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is
   (a) 200 : 1 (b) 150 : 1 (c) 100 : 1 (d) 50 : 1

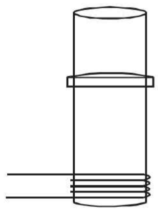
PASSAGE 3

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ. *(JEE Adv. 2013)*

6. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is
   (a) $\frac{BR}{4}$ (b) $\frac{BR}{2}$ (c) BR (d) 2BR
7. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change, is
   (a) $-\gamma BQR^2$ (b) $-\gamma\frac{BQR^2}{2}$ (c) $\gamma\frac{BQR^2}{2}$ (d) γBQR^2

H Assertion & Reason Type Questions

1. **Statement-1 :** A vertical iron rod has coil of wire wound over it at the bottom end. An alternating current flows in the coil. The rod goes through a conducting ring as shown in the figure. The ring can float at a certain height above the coil.

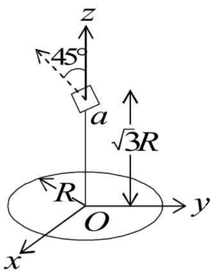


   Statement-2 : In the above situation, a current is induced in the ring which interacts with the horizontal component of the magnetic field to produce an average force in the upward direction. *(2007)*
   (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
   (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
   (c) Statement-1 is True, Statement-2 is False
   (d) Statement-1 is False, Statement-2 is True.

I Integer Value Correct Type

1. A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is *(2011)*

2. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side $a(a<<R)$ having two turns is placed with its centre at $z = \sqrt{3}R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is *(2012)*



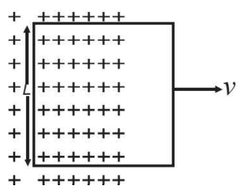
18. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max} / I_{min}) drawn from the battery is *(JEE Adv. 2016)*

Section-B JEE Main / AIEEE

1. The power factor of an AC circuit having resistance (R) and inductance (L) connected in series and an angular velocity ω is

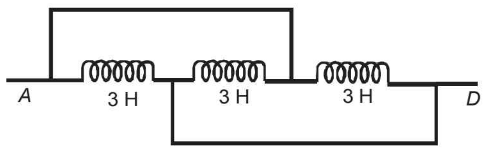
(a) $R/\omega L$ (b) $R/(R^2 + \omega^2 L^2)^{1/2}$
(c) $\omega L/R$ (d) $R/(R^2 - \omega^2 L^2)^{1/2}$

2. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in figure. The induced emf is [2002]



(a) zero (b) RvB
(c) vBL/R (d) vBL

3. The inductance between A and D is [2002]



(a) 3.66 H (b) 9 H (c) 0.66 H (d) 1 H.

4. In a transformer, number of turns in the primary coil are 140 and that in the secondary coil are 280. If current in primary coil is 4 A, then that in the secondary coil is [2002]

(a) 4 A (b) 2 A (c) 6 A (d) 10 A.

5. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon [2003]

(a) the rates at which currents are changing in the two coils
(b) relative position and orientation of the two coils
(c) the materials of the wires of the coils
(d) the currents in the two coils

6. When the current changes from +2 A to -2A in 0.05 second, an e.m.f. of 8 V is induced in a coil. The coefficient of self-induction of the coil is [2003]

(a) 0.2 H (b) 0.4 H (c) 0.8 H (d) 0.1 H

7. In an oscillating LC circuit the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is [2003]

(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$ (c) $\frac{Q}{\sqrt{2}}$ (d) Q

8. The core of any transformer is laminated so as to [2003]

(a) reduce the energy loss due to eddy currents
(b) make it light weight
(c) make it robust and strong
(d) increase the secondary voltage

9. Alternating current can not be measured by D.C. ammeter because [2004]

(a) Average value of current for complete cycle is zero
(b) A.C. Changes direction
(c) A.C. can not pass through D.C. Ammeter
(d) D.C. Ammeter will get damaged.

10. In an LCR series a.c. circuit, the voltage across each of the components, L, C and R is 50V. The voltage across the LC combination will be [2004]

(a) 100 V (b) $50\sqrt{2}$ V (c) 50 V (d) 0 V (zero)

11. A coil having n turns and resistance $R\Omega$ is connected with a galvanometer of resistance $4R\Omega$. This combination is moved in time t seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is [2004]

(a) $-\frac{(W_2 - W_1)}{Rnt}$ (b) $-\frac{n(W_2 - W_1)}{5\ Rt}$
(c) $-\frac{(W_2 - W_1)}{5\ Rnt}$ (d) $-\frac{n(W_2 - W_1)}{Rt}$

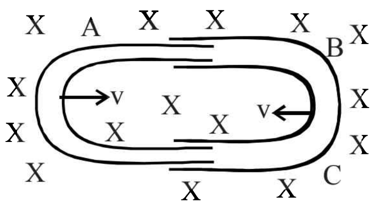
12. In a uniform magnetic field of induction B a wire in the form of a semicircle of radius r rotates about the diameter of the circle with an angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R, the mean power generated per period of rotation is

(a) $\frac{(B\pi r\omega)^2}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$
(c) $\frac{B\pi r^2\omega}{2R}$ (d) $\frac{(B\pi r\omega^2)^2}{8R}$ [2004]

13. In a LCR circuit capacitance is changed from C to $2\ C$. For the resonant frequency to remain unchanged, the inductance should be changed from L to **[2004]**
(a) $L/2$ (b) $2L$ (c) $4L$ (d) $L/4$

14. A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 radians per second. If the horizontal component of earth's magnetic field is 0.2×10^{-4}T, then the e.m.f. developed between the two ends of the conductor is **[2004]**
(a) 5mV (b) 50μV (c) 5μV (d) 50mV

15. One conducting U tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure . If each tube moves towards the other at a constant speed v, then the emf induced in the circuit in terms of B, l and v where l is the width of each tube, will be **[2005]**



(a) $-Blv$ (b) Blv
(c) $2\ Blv$ (d) zero

16. The self inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of **[2005]**
(a) 8μF (b) 4μF (c) 2μF (d) 1μF

17. The phase difference between the alternating current and emf is $\frac{\pi}{2}$. Which of the following cannot be the constituent of the circuit? **[2005]**
(a) R, L (b) C alone (c) L alone (d) L, C

18. A circuit has a resistance of 12 ohm and an impedance of 15 ohm. The power factor of the circuit will be **[2005]**
(a) 0.4 (b) 0.8 (c) 0.125 (d) 1.25

19. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in **[2005]**
(a) 0.1 s (b) 0.05 s (c) 0.3 s (d) 0.15 s

20. Which of the following units denotes the dimension $\frac{ML^2}{Q^2}$, where Q denotes the electric charge? **[2006]**
(a) Wb/m^2 (b) Henry (H)
(c) H/m^2 (d) Weber (Wb)

21. In a series resonant LCR circuit, the voltage across R is 100 volts and $R = 1$ kΩ with C = 2μF. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is **[2006]**
(a) 2.5×10^{-2} V (b) 40 V
(c) 250 V (d) 4×10^{-3} V

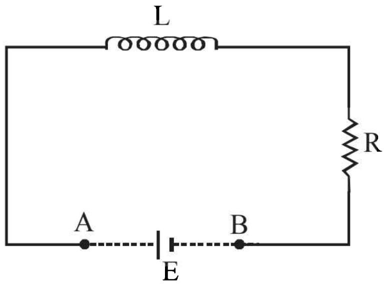
22. In an AC generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω in a magnetic field B. The maximum value of emf generated in the coil is **[2006]**
(a) N.A.B.R.ω (b) N.A.B
(c) N.A.B.R. (d) N.A.B.ω

23. The flux linked with a coil at any instant 't' is given by

$$\phi = 10t^2 - 50t + 250$$

The induced emf at $t = 3s$ is **[2006]**
(a) –190 V (b) –10 V (c) 10 V (d) 190 V

24. An inductor (L = 100 mH), a resistor (R = 100 Ω) and a battery (E = 100 V) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is **[2006]**



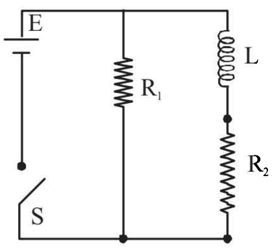
(a) 1/eA (b) eA (c) 0.1 A (d) 1 A

25. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by **[2007]**
(a) $P = \sqrt{2}E_0 I_0$ (b) $P = \frac{E_0 I_0}{\sqrt{2}}$
(c) $P =$ zero (d) $P = \frac{E_0 I_0}{2}$

26. An ideal coil of 10H is connected in series with a resistance of 5Ω and a battery of 5V. 2second after the connection is made, the current flowing in ampere in the circuit is **[2007]**
(a) $(1-e^{-1})$ (b) $(1-e)$ (c) e (d) e^{-1}

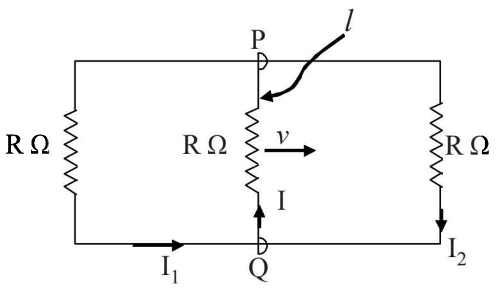
27. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area A = 10 cm^2 and length = 20 cm. If one of the solenoid has 300 turns and the other 400 turns, their mutual inductance is **[2008]**
($\mu_0 = 4\pi\times10^{-7}$ Tm A^{-1})
(a) $2.4\pi\times10^{-5}$ H (b) $4.8\pi\times10^{-4}$ H
(c) $4.8\pi\times10^{-5}$ H (d) $2.4\pi\times10^{-4}$ H

28. An inductor of inductance L = 400 mH and resistors of resistance R_1 = 2Ω and R_2 = 2Ω are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is : **[2009]**



(a) $\frac{12}{t}e^{-3t}$ V (b) $6\left(1-e^{-t/0.2}\right)$ V
(c) $12e^{-5t}$ V (d) $6e^{-5t}$ V

29. A rectangular loop has a sliding connector PQ of length l and resistance R Ω and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are **[2010]**



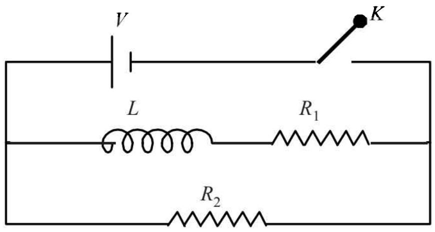
(a) $I_1 = -I_2 = \frac{Blv}{6R},\ I = \frac{2Blv}{6R}$

(b) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

(c) $I_1 = I_2 = I = \frac{Blv}{R}$

(d) $I_1 = I_2 = \frac{Blv}{6R},\ I = \frac{Blv}{3R}$

30. In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is **[2010]**



(a) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(b) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = \infty$

(c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

(d) $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

31. In a series LCR circuit $R = 200\Omega$ and the voltage and the frequency of the main supply is 220V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is **[2010]**

(a) 305 W (b) 210 W
(c) Zero W (d) 242 W

32. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} NA^{-1} m^{-1} due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is: **[2011]**

(a) 0.75 mV (b) 0.50 mV
(c) 0.15 mV (d) 1 mV

33. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is: **[2011]**

(a) $\frac{\pi}{4}\sqrt{LC}$ (b) $2\pi\sqrt{LC}$
(c) $\sqrt{LC}$ (d) $\pi\sqrt{LC}$

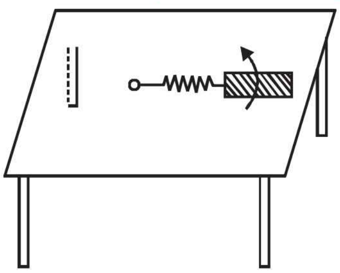
34. A resistor 'R' and 2μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. ($\log_{10}2.5 = 0.4$) **[2011]**

(a) $1.7 \times 10^5\,\Omega$ (b) $2.7 \times 10^6\,\Omega$
(c) $3.3 \times 10^7\,\Omega$ (d) $1.3 \times 10^4\,\Omega$

35. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; It is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to : **[2012]**

(a) developement of air current when the plate is placed
(b) induction of electrical charge on the plate
(c) shielding of magnetic lines of force as aluminium is a paramagnetic material.
(d) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.

36. A metallic rod of length 'ℓ' is tied to a string of length 2ℓ and made to rotate with angular speed w on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is **[JEE Main 2013]**



(a) $\frac{2B\omega\ell^2}{2}$

(b) $\frac{3B\omega\ell^2}{2}$

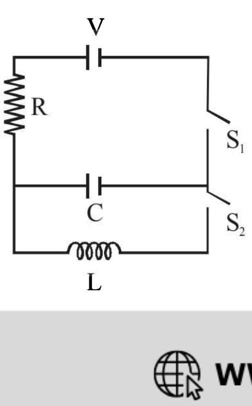
(c) $\frac{4B\omega\ell^2}{2}$

(d) $\frac{5B\omega\ell^2}{2}$

37. A circular loop of radius 0.3 cm lies parallel to amuch bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is **[JEE Main 2013]**

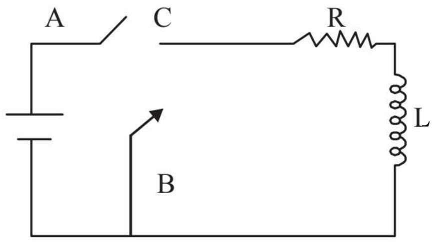
(a) 9.1×10^{-11} weber (b) 6×10^{-11} weber
(c) 3.3×10^{-11} weber (d) 6.6×10^{-9} weber

38. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is Capacitive time constant). Which of the following statement is correct ? **[JEE Main 2013]**



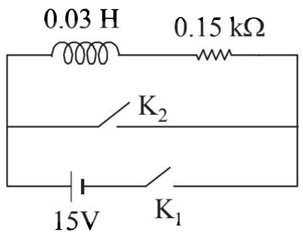
(a) Work done by the battery is half of the energy dissipated in the resistor
(b) At $t = \tau$, $q = CV/2$
(c) At $t = 2\tau$, $q = CV(1 - e^{-2})$
(d) At $t = 2\tau$, $q = CV(1 - e^{-1})$

39. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to: **[JEE Main 2014]**



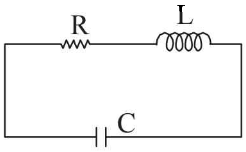
(a) $\frac{e}{1-e}$ (b) 1
(c) –1 (d) $\frac{1-e}{e}$

40. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ ms, the current in the circuit will be : $(e^5 \cong 150)$ **[JEE Main 2015]**

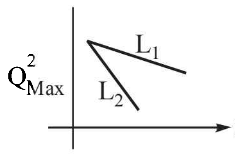


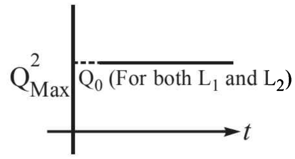
(a) 6.7 mA (b) 0.67 mA
(c) 100 mA (d) 67 mA

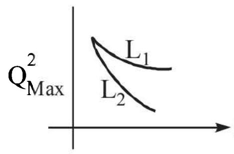
41. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below :

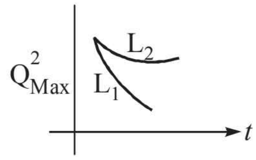


If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time(t) for two different values L_1 and L_2 $(L_1 > L_2)$ of L then which of the following represents this graph correctly ? (*plots are schematic and not drawn to scale*) **[JEE Main 2015]**

(a) 

(b) 

(c) 

(d) 

42. Two coaxial solenoids of different radius carry current I in the same direction. $\vec{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then : **[JEE Main 2015]**
(a) $\vec{F_1}$ is radially inwards and $\vec{F_2} = 0$
(b) $\vec{F_1}$ is radially outwards and $\vec{F_2} = 0$
(c) $\vec{F_1} = \vec{F_2} = 0$
(d) $\vec{F_1}$ is radially inwards and $\vec{F_2}$ is radially outwards

43. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to :
(a) 0.044 H (b) 0.065 H
(c) 80 H (d) 0.08 H

15 Electromagnetic Induction and Alternating Current

Section-A : JEE Advanced/ IIT-JEE

A 1. 0.3×10^{-4} sec, 10 A 2. Left to right

B 1. T 2. F 3. T

C 1. (b) 2. (d) 3. (c) 4. (d) 5. (b) 6. (b) 7. (d)
8. (a) 9. (d) 10. (b) 11. (a) 12. (c) 13. (a) 14. (b)
15. (d) 16. (b)

D 1. (a, b, c) 2. (d) 3. (a, c, d) 4. (b) 5. (a, b, c, d) 6. (b) 7. (a, c)
8. (a, c) 9. (a) 10. (c, d) 11. (a,d) 12. (a,b)

E 1. Clockwise 2. 1mV 3. Yes, opposite direction of A 4. 0.02 m/s, clockwise direction

5. (i) $\frac{1}{2}\frac{Br^2\omega}{R}$ (ii) anticlockwise 6. (i) $V = \left(\frac{R+2\lambda x}{Bd}\right) I,\ F = BId + \frac{2\lambda m I^2}{(Bd)^2}(R+2\lambda x)$, (ii) $\left[1+\frac{2\lambda m I(R+2\lambda x)}{B^3 d^3}\right]^{-1}$

7. (a) –5V, 24.5 W (b) (i) 0.6 amp. (ii) 1.386×10^{-3} sec., 4.5×10^{-4} J 8. $\frac{7}{22}$ amp, $\frac{6}{22}$ amp, $\frac{1}{22}$ amp

9. 1 m/s, 0.47 Ω, 0.3 Ω 10. (a) $\frac{Br^2\omega}{2}$ (b) $I = \frac{B\omega r^2}{2R}\left[1-e^{-\left(\frac{R}{L}\right)t}\right]$, $\tau = \frac{B^2 r^4 w}{4R} + \frac{mgr}{2}\cos\omega t$ 11. 3.466 sec

12. (i) $\frac{mgR}{B^2L^2}$ (ii) $\frac{g}{2}$ 13. (a) $\frac{B_0 a v(t)}{R}$, anticlockwise (b) $-\frac{B_0^2 a^2 v(t)}{R}$, upward

(c) $\frac{mgR}{B_0^2 a^2}\left[1-e^{\frac{-B_0^2 a^2 t}{mR}}\right]$; $\frac{mgr}{B_0^2 a^2}$

14. $12e^{-5t}V$, $3e^{-10t}A$, clockwise

15. (a) P to Q (b) $IbB_0(3\hat{k}-4\hat{i})$ (c) $I = \frac{mg}{6aB_0}$ 16. (a) $\frac{d\phi}{dt} = iR + L\frac{di}{dt}$ (b) $\frac{1}{R}\left[\frac{\mu_0 I_0 \ell}{2\pi}\log_e 2\right] - \frac{L}{R}i_1$ (c) $\frac{T}{2\log_e 2}$

17. (a) $\frac{\mu_0 a I_0 \omega^2 C \ln(2)}{\pi}$ 18. 20A, $\frac{\pi}{4}$ 19. $\frac{4}{3}(r_1 - r_2)$ 20. $I = \frac{\mu_0 n a^2 L d i_0 \omega \cos\omega t}{2\rho R}$

F 1. A-r, s, t; B-q, r, s, t; C-p, q; D-q, r, s, t

G 1. (b) 2. (d) 3. (c) 4. (a) 5. (b) 6. (b) 7. (b)

H 1. (a)

I 1. 4 2. 7 3. 8

Section-B : JEE Main/ AIEEE

1. (b) 2. (d) 3. (d) 4. (b) 5. (b) 6. (d) 7. (c)
8. (a) 9. (a) 10. (d) 11. (b) 12. (b) 13. (a) 14. (b)
15. (c) 16. (d) 17. (a) 18. (b) 19. (a) 20. (b) 21. (c)
22. (d) 23. (b) 24. (a) 25. (c) 26. (a) 27. (d) 28. (c)
29. (b) 30. (b) 31. (d) 32. (c) 33. (a) 34. (b) 35. (d)
36. (d) 37. (a) 38. (c) 39. (c) 40. (b) 41. (c) 42. (c)
43. (b)

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. The coil is broken into two identical coils.

$$L_{eq} = \frac{L/2 \times L/2}{L/2 + L/2} = \frac{L}{4} = 0.45 \times 10^{-4}\text{ H},$$

$$R_{eq} = \frac{R/2 \times R/2}{R/2 + R/2} = \frac{R}{4} = 1.5\Omega$$

$$\text{Time constant} = \frac{L_{eq}}{R_{eq}} = \frac{0.45 \times 10^{-4}}{1.5} = 0.3 \times 10^{-4}\text{s}$$

$$\text{Steady current } I = \frac{E}{R_{eq}} = \frac{15}{1.5} = 10\text{ A}.$$

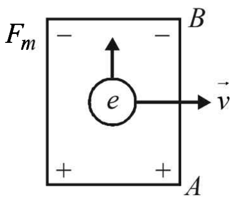
2. **NOTE:** As the source is switched off, the current decreases to zero. The induced current opposes the cause as per Lenz's law. Therefore, the induced current will direct from left to right.

B. True/ False

1. **True.** A copper wire consists of billions and billions of free electrons. When the wire is at rest, the average velocity of each electron is zero. But when the wire is in motion, the electrons have a net velocity in the direction of motion.
NOTE : A charged particle moving in a magnetic field experiences a force given by $\vec{F} = q(\vec{v} \times \vec{B})$.
Here also each electron experiences a force and therefore, electrons will move towards one end creating an emf between the two ends of a straight copper wire.

2. **NOTE :** For induced emf to develop in a coil, the magnetic flux through it must change.
But in this case the number of magnetic lines of force through the coil is not changing. Therefore the statement is false.

3. **NOTE :** When conduction rod AB moves parallel to x-axis in a uniform magnetic field pointing in the positive z-direction, then according to Fleming's left hand rule, the electrons will experience a force towards B. Hence, the end A will become positive.



C. MCQs with ONE Correct Answer

1. **(b)** The current induced will be

$$i = \frac{|e|}{R} \Rightarrow i = \frac{1}{R}\frac{d\phi}{dt} \quad \text{But } i = \frac{dq}{dt}$$

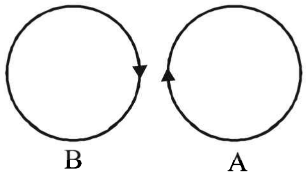
$$\Rightarrow \frac{dq}{dt} = \frac{1}{R}\frac{d\phi}{dt} \Rightarrow \int dq = \frac{1}{R}\int d\phi \Rightarrow q = \frac{BA}{R}$$

2. **(d)** Induced emf produced across MNQ will be same as the induced emf produced in straight wire MQ.

$\therefore \quad e = Bv\ell = Bv \times 2R$ with Q at higher potential.

3. **(c)** When the current in the loop A increases, the magnetic lines of force in loop B also increases as loop A is placed near loop B. This induces an emf in B in such a direction that current flows opposite in B (as compared to A).

Since currents are in opposite direction, the loop B is repelled by loop A.



4. **(d)** **KEY CONCEPT :** Using $I = I_0(1 - e^{-t/\tau})$

But $\quad I_0 = \frac{V}{R} \quad$ and $\quad \tau = \frac{L}{R}$

$$\therefore \quad I = \frac{V}{R}(1 - e^{-Rt/L}) = \frac{12}{6}\left[1 - e^{-6t/8.4\times10^{-3}}\right]$$

$= 1$ (given)

$\therefore \quad t = 0.97 \times 10^{-3}\text{ s} \approx 1\text{ms}$

5. **(b)** $\oint \vec{E}.\vec{d\ell} = \frac{d\phi}{dt} = \frac{d}{dt}(\vec{B}.\vec{A}) = \frac{d}{dt}(BA\cos 0°) = A\frac{dB}{dt}$

$$\Rightarrow \quad E(2\pi r) = \pi a^2 \frac{dB}{dt} \; for\; r \geq a$$

$$\Rightarrow \quad E = \frac{a^2}{2r}\frac{dB}{dt} \Rightarrow E \propto \frac{1}{r}$$

6. **(b)** **KEY CONCEPT :** The magnetic field at the centre of the coil

$B(t) = \mu_0 n I_1$.

As the current increases, B will also increase with time till it reaches a maximum value (when the current becomes steady).

The induced emf in the ring

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(\vec{B}.\vec{A}) = -A\frac{d}{dt}(\mu_0 n I_1)$$

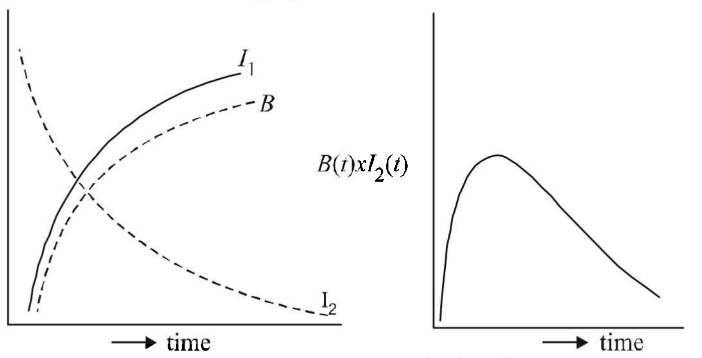
$\therefore$ The induced current in the ring

$$I_2(t) = \frac{|e|}{R} = \frac{\mu_0 n A}{R}\frac{dI_1}{dt}$$

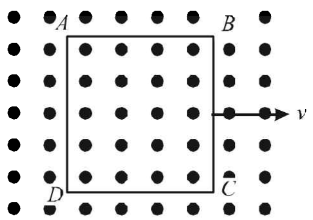
[**NOTE :** $\frac{dI_1}{dt}$ decreases with time and hence I_2 also decreases with time.]

Where $I_1 = I_{max}(1 - e^{-t/\tau})$

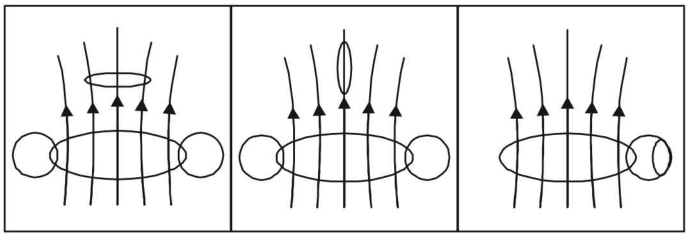
The relevant graphs are



7. **(d)** **NOTE :** Electric field will be induced, as $ABCD$ moves, in both AD and BC. The metallic square loop moves in its own plane with velocity v. A uniform magnetic field is imposed perpendicular to the plane of the square loop. AD and BC are perpendicular to the velocity as well as perpendicular to applied.



8. **(a)** Clearly the flux linkage is maximum in case (a) due to the spatial arrangement of the two loops.



9. **(d)** When switch S is closed, a magnetic field is set-up in the space around P. The field lines threading Q increases in the direction from right to left. According to Lenz's law, I_{Q_1} will flow so as to oppose the cause and flow in anticlockwise direction as seen by E. Reverse is the case when S is opened. I_{Q_2} will be clockwise.

10. **(b)** **KEY CONCEPT :**

$$P=\frac{E^2}{R}=\frac{\pi r^2}{\rho \ell}\left(\frac{d\phi}{dt}\right)^2=\frac{\pi r^2}{\rho \ell}\left[\frac{d}{dt}(NBA)^2\right]$$

$$=\frac{\pi r^2}{\rho \ell}N^2A^2\left(\frac{dB}{dt}\right)^2 \Rightarrow P \propto \frac{N^2 r^2}{\ell}$$

Case 1 : $P_1 \propto \frac{N^2 r^2}{\ell}$, **Case 2 :** $P_2 \propto \frac{(4N)^2 (r/2)^2}{4\ell}$

NOTE : When we decrease the radius of the wire, its length increases but volume remains the same]

$$\Rightarrow \frac{P_1}{P_2}=\frac{1}{1}$$

$\therefore$ Power remains the same.

11. **(a)** **NOTE :** Since current leads emf (as seen from the graph), therefore, this is an $R-C$ circuit.

$$\tan\phi=\frac{X_C-X_L}{R}$$

Here $\phi = 45°$

$\therefore$ $X_C = R$ [$X_L = 0$ as there is no inductor]

$$\frac{1}{\omega C}=R \Rightarrow RC\omega = 1$$

$$\therefore \quad RC=\frac{1}{100}s^{-1}$$

12. **(c)** **KEY CONCEPT :** Initially, ϕ_B increases as magnet approaches the solenoid

$\therefore$ $\varepsilon = -$ ve and increasing in magnitude. When magnet is moving inside the solenoid, increase in ϕ_B slow down and finally ϕ_B starts decreasing

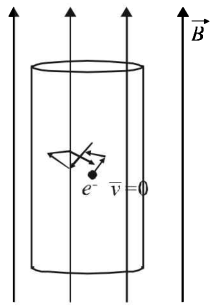
$\therefore$ emf is positive and increasing.

Only graph (c) shows these characteristic.

13. **KEY CONCEPT :** For a current to induce in cylindrical conducting rod,

(a) the conducting rod should cut magnetic lines of force which will happen only when the conducting rod is moving. Since conducting rod is at rest, no current will be induced.

(b) the magnitude/direction of the magnetic field changes. A changing magnetic field will create an electric field which can apply force on the free electrons of the conducting rod and a current will get induced. But since the magnetic field is constant, no current will be induced.



14. **(b)** **KEY CONCEPT :**

Time constant of $R-C$ circuit is $\tau = R_{eq}C_{eq}$

(i) $\tau_1=(2+1)(2+4)=18\,\mu s$

(ii) $\tau_2=\left(\frac{2\times1}{2+1}\right)\left(\frac{2\times4}{2+4}\right)=\frac{8}{9}\mu s$

(iii) $\tau_3=\left(\frac{2\times1}{2+1}\right)\times(4+2)=4\mu s$

15. **(d)** The magnetic field is increasing in the downward direction. Therefore, according to Lenz's law the current I_1 will flow in the direction ab and I_2 in the direction dc.

16. **(b)** $I_{rms}=\frac{V_{rms}}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}$

As ω increases, I_{rms} increases. Therefore the bulb glows brighter.

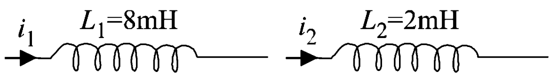
D. MCQs with ONE or MORE THAN ONE Correct

1. **(a,b,c)**

$\frac{1}{RC}$, R/L and $1/\sqrt{LC}$ have the dimensions of frequency.

2. **(d)** **NOTE :** Since the rate of change of magnetic flux is zero, hence there will be no net induced emf and hence no current flowing in the loop.

3. **(a, c, d)**



Rate of change of current $=\frac{di_1}{dt}=m(say)$

Induced emf $V_1=-L_1\frac{di_1}{dt}=-8\times10^{-3}\times m$

$\therefore \frac{V_2}{V_1}=\frac{1}{4}$

Power $P=V_1 i_1=8\times10^{-3}\times m\times i_1$

Rate of change of current $=\frac{di_2}{dt}=m\text{(given)}$

Induced emf, $V_2=-L_2\frac{di_2}{dt}=-2\times10^{-3}\times m$

Power $P=V_2 i_2=2\times10^{-3}\times m\times i_2$

Since Power is equal

$\therefore\ 8\times10^{-3}\times mi_1=2\times10^{-3}\ mi_2$

$\Rightarrow \frac{i_1}{i_2}=\frac{1}{4}$ ...(i)

Energy $W_1=\frac{1}{2}L_1 i_1^2=\frac{1}{2}\times8\times10^{-3}\times i_1^2$

$\therefore \frac{W_2}{W_1}=\frac{10^{-3}\times i_2^2}{4\times10^{-3}\times i_1^2}=\frac{1}{4}\times4\times4=4$ [from (i)]

4. **(b)** **KEY CONCEPT :** The magnetic field due to a current flowing in a wire of finite length is given by

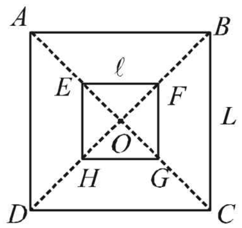
$$B=\frac{\mu_0 I}{4\pi R}(\sin\alpha+\sin\beta)$$

Applying the above formula for AB for finding the field at O, we get

$$B=\frac{\mu_0 I_1}{4\pi(L/2)}(\sin45°+\sin45°)=\frac{\mu_0 I_1}{\sqrt{2}\pi L}$$

acting perpendicular to the plane of paper upwards

$\therefore$ The total magnetic field due to current flowing through $ABCD$ is



$$B=4B_1=\frac{4\mu_0 I_1}{\sqrt{2}\pi L}=\frac{2\sqrt{2}\mu_0 I_1}{\pi L}$$

The total flux passing through the square EFGH

$$\phi_2=B\times\ell^2=\frac{2\sqrt{2}\mu_0 I_1}{\pi L}\times\ell^2 \quad ...(i)$$

[$\because \ell > L$ and therefore, B can be assumed constant for ℓ^2]

The flux through small square loop is directly proportional to the current passing through big square loop.

$\therefore \phi_2\propto I_1\Rightarrow\phi=MI_1$ where M = Mutual Conductance

$$\therefore M_2=\frac{\phi_2}{I_1}=\frac{\frac{2\sqrt{2}\mu_0 I_1}{\pi L}\times\ell^2}{I_1}=\frac{2\sqrt{2}\mu_0}{\pi L}\times\ell^2$$

$$\Rightarrow M\propto\frac{\ell^2}{L}.$$

5. **(a, b, c, d)**

(a) $L=\frac{\phi}{i}$ or henry $=\frac{\text{weber}}{\text{ampere}}$

(b) $e=-L\left(\frac{di}{dt}\right)$

$\therefore L=-\frac{e}{(di/dt)}$ or henry $=\frac{\text{volt}-\text{second}}{\text{ampere}}$

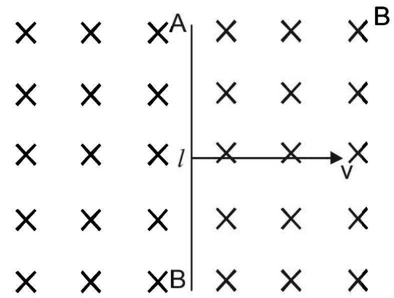
(c) $U=\frac{1}{2}Li^2$

$\therefore L=\frac{2U}{i^2}$ or henry $=\frac{\text{joule}}{(\text{ampere})^2}$

(d) $U=\frac{1}{2}Li^2=i^2Rt$

$\therefore L=R.t$ or henry = ohm-second.

6. **(b)**



A motional emf, e = Blv is induced in the rod. Or we can say a potential difference is induced between the two ends of the rod AB, with A at higher potential and B at lower potential. Due to this potential difference, there is an electric field in the rod.

7. **(a,c)** We know that $Z=\sqrt{R^2+\left(\frac{1}{WC}\right)^2}$

The capacitance in case B is four times the capacitance in case A

$\therefore$ Impedance in case B is less then that of case A ($Z_B<Z_A$)

Now $I=\frac{V}{Z}$

$\therefore I_R^A<I_R^B$. option (a) is correct.

$\therefore V_R^A<V_R^B$.

$\Rightarrow V_C^A>V_C^B$

[$\because$ If V is the applied potential difference access series R-C circuit then $V=\sqrt{V_R^2+V_C^2}$]

$\therefore$ (c) is the correct option.

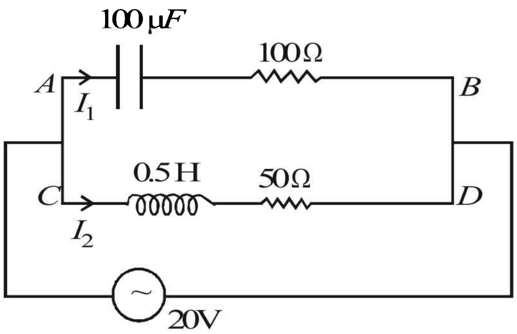
8. (a,c) Impedance across AB

$$Z_1 = \sqrt{X_c^2 + R_1^2} = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R_1^2}$$

$$= \sqrt{(100)^2 + (100)^2} = 100\sqrt{2}$$

$\therefore \quad I_1 = \frac{V}{Z_1} = \frac{20}{100\sqrt{2}}$ [leads emf by ϕ_1]

where $\cos\phi_1 = \frac{R}{Z_1} = \frac{100}{100\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

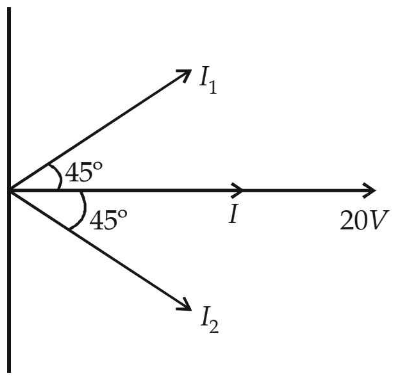


Impedance across CD is

$$Z_2 = \sqrt{X_L^2 + R_2^2} = \sqrt{(\omega L)^2 + R_2^2}$$

$$= \sqrt{(0.5 \times 100)^2 + (50)^2} = 50\sqrt{2}\ \Omega$$

$\therefore \quad I_2 = \frac{V}{Z_2} = \frac{20}{50\sqrt{2}}$ [leads emf by ϕ_2]



where $\cos\phi_2 = \frac{R}{Z_2} = \frac{50}{50\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \phi_2 = 45°$

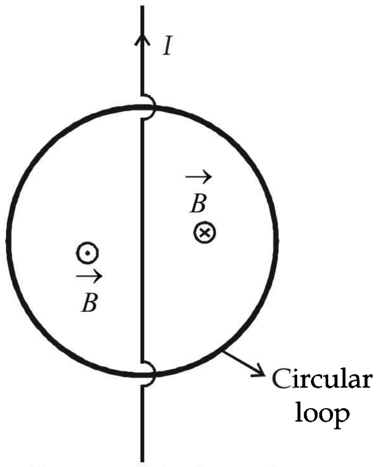
$\therefore$ The current I from the circuit is

$I = I_1 + I_2 \simeq 0.3$ A

9. (a) Emf will be induced in the circular wire loop when flux through it changes with time.

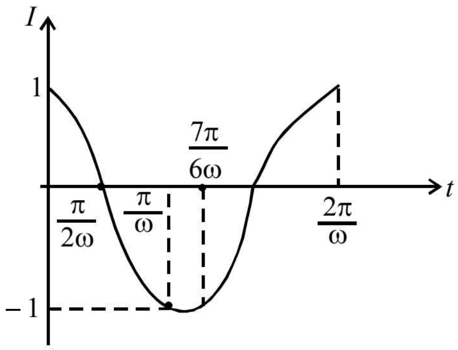
$$e = -\frac{\Delta\phi}{\Delta t}$$

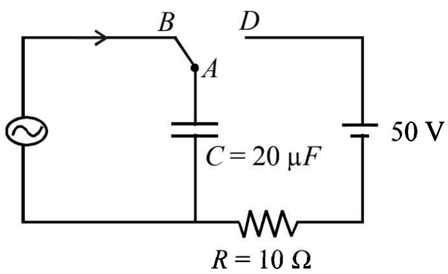
when the current is constant, the flux changing through it will be zero.



When the current is decreasing at a steady rate then the change in the flux (decreasing inwards) on the right half of the wire is equal to the change in flux (decreasing outwards) on the left half of the wire such that $\Delta\phi$ through the circular loop is zero.

10. (c, d) $I = \cos 500\,t$





Till $t = \frac{7\pi}{6\omega}$, the charge will be maximum at $\frac{\pi}{2\omega}$

$$Q' = \int_0^{\pi/2\omega} \cos 500t\ dt = \left[\frac{\sin 500t}{500}\right]_0^{\pi/2\omega}$$

$$= \frac{1}{500}\sin\left(500 \times \frac{\pi}{2 \times 500}\right) = \frac{1}{500}\text{C}$$

$\therefore$ (a) is incorrect

From the graph it is clear that just before $t = \frac{7\pi}{6\omega}$, the current is in anticlockwise direction.

$\therefore$ (b) is incorrect

At $t=\frac{7\pi}{6\omega}$, the charge on the upper plate of capacitor is

$$\int_0^{\frac{7\pi}{6\omega}} \cos 500t\, dt = \frac{1}{500}\sin\left(500\times\frac{7\pi}{6\times 500}\right)$$

$$=-\frac{1}{500}\times\frac{1}{2}=-10^{-3}C$$

Now applying KVL (when A is just connected to D)

$$50+\frac{10^{-3}}{20\times10^{-6}}-i\times10=0 \Rightarrow i=10\text{ A}$$

$\therefore$ (c) is the correct option.

The maximum charge on C is $Q=CV=20\times10^{-6}\times50$ $=10^{-3}$C

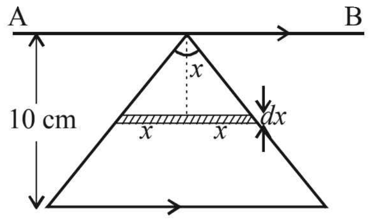
Therefore, the total charge flown $=2\times10^{-3}$ C

$\therefore$ (d) is the correct option.

11. (a, d)

The flux passing through the triangular wire if i current flows through the inifinitely long conducting wire

$$=\int_0^{0.1}\frac{\mu_0 i}{2\pi x}\times 2\pi dx$$



$$\phi=\frac{\mu_0 i}{10\pi}=Mi$$

$$\therefore M=\frac{\mu_0}{10\pi} \qquad \frac{dI}{dt}=10AS^{-1}$$

Induced emf in the wire $=M\frac{di}{dt}=\frac{\mu_0}{10\pi}\times10=\frac{\mu_0}{\pi}$

As the current in the triangular wire is decreasing the induced current in AB is in the same direction as the current in the hypotenuse of the triangular wire. Therefore force will be repulsive.

12. (a, b) $i=\frac{BLv}{R}$ – (i) [Counter clockwise direction while entering, Zero when completely inside and clockwise while exiting]

$F=iLB=\frac{B^2L^2v}{R}$ – (ii) [Toward left while entering and exiting and zero when completely inside]

$$\therefore \quad -mV\frac{dv}{dx}=\frac{B^2L^2v}{R}$$

$$\therefore \quad \int_{v_0}^{v} dV=-\frac{B^2L^2}{mR}\int_0^x dx$$

$$V-V_0=-\frac{B^2L^2}{mR}x$$

$$\therefore \quad V=V_0-\frac{B^2L^2x}{mR} \quad \text{–(iii)}$$

[V decreases from x=0 to x=L, remains constant for x = L to x= 3L again decreases from x=3L to x=4L]

From (i) and (iii)

$$i=\frac{BL}{R}\left[V_0-\frac{B^2L^2x}{mR}\right]$$

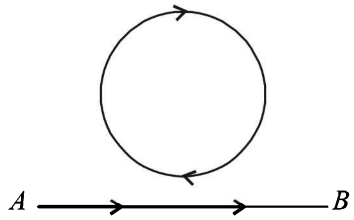
[i decreases from x=0 to x=L i becomes zero from x = L to x = 3L i changes direction and decreases from x = 3L to x = 4L]

These characteristics are shown in graph (a) and (b) only.

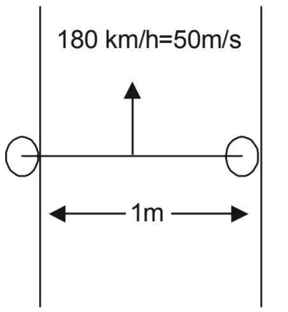
E. Subjective Problems

1. The magnetic lines of force due to current flowing in wire AB is shown.

NOTE : As the current increases, the number of magnetic lines of force passing through the loop increases in the outward direction. To oppose this change, the current will flow in the clockwise direction.

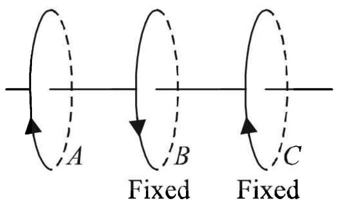


2. KEY CONCEPT : This is based on motional emf.



$$e=vB\ell=50\times0.2\times10^{-4}\times1=10^{-3}\text{ volt}=1\text{ milli volt}$$

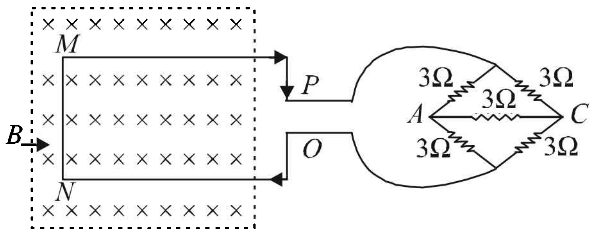
3.

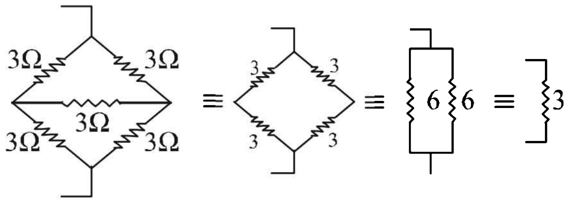


NOTE : When the coil A moves towards B, the number of magnetic lines of force passing through B changes. Therefore, an induced emf and hence induced current is produced in B.

The direction of current in B will be such as to oppose the field change in B and therefore, will be in the opposite direction of A.

4.





NOTE : The network behaves like a balanced wheatstone bridge.

The free electrons in the portion MN of the rod have a velocity v in the right direction. Applying Fleming's left hand rule, we find that the force on electron will be towards N. Hence, M will be + ve and N will be negative. Current will flow in clockwise direction.

The induced emf developed is given by

$e = vB\ell = v \times 2 \times 0.1 = 0.2v$ ...(i)

Now, $e = IR$

$e = 10^{-3} \times 4 = 4 \times 10^{-3}$ amp ...(ii)

From (i) and (ii),

$0.2\,v = 4 \times 10^{-3}$

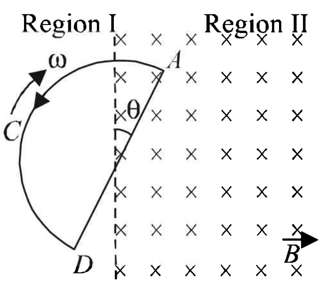
$\therefore \quad v = \dfrac{4\times 10^{-3}}{0.2} = 0.02$ m/s

5. (i) Induced emf

$$E = -\frac{d\phi}{dt} = -\frac{d}{dt}(B \times A)$$

$$= -\frac{d}{dt}\left[B\left(\frac{1}{2}r^2\theta\right)\right] = -\frac{1}{2}Br^2\frac{d\theta}{dt} = -\frac{1}{2}Br^2\omega$$

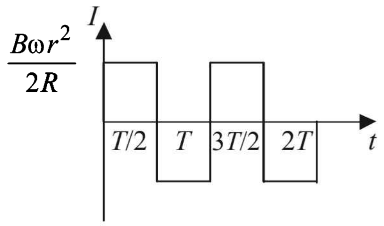
$$\therefore \quad I = \frac{E}{R} = -\frac{1}{2}\frac{Br^2\omega}{R} \Rightarrow |I| = \frac{1}{2}\frac{Br^2\omega}{R}$$



(ii) The loop is entering in the magnetic field and hence magnetic lines of force passing through the loop is increasing in the downward direction. Therefore, current will flow in the loop in such a direction which will oppose the change. The current will flow in the anti-clockwise direction.

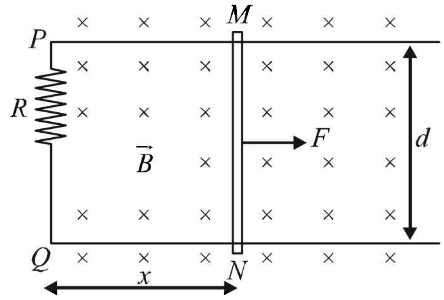
(iii) Graph between induced emf and period of rotation:
For first half rotation, ($t = T/2$), when the loop enters the field, the current is in anticlockwise direction.

Magnitude of current remains constant at $I = B\omega r^2/2R$.



For next half rotation, when the loop comes out of the field, current of the same magnitude is set up clockwise. Anticlockwise current is supposed to be positive. The I-t graph is shown in the figure for two periods of rotation.

6. (i) A variable force F is applied to the rod MN such that as the rod moves in the uniform magnetic field a constant current flows through R. Consider the loop $MPQN$.
Let MN be at a distance x from PQ.
Length of rails in loop = $2x$



$\therefore$ Resistance of rails in loop = $2x\lambda$

$\therefore$ Total resistance of loop = $R + 2\lambda x$

Induced emf = Bvd

$\therefore$ Induced current $(I) = \dfrac{Bvd}{R + 2\lambda x}$

So for constant I,

$v = \dfrac{(R+2\lambda x)}{Bd}I$ ...(i)

Furthermore, as due to induced current I the wire will experience a force $F_M = BId$ opposite to its motion, the equation of motion of the wire will be

$F - F_M = ma$ i.e., $F = F_M + ma$

But as here $F_M = BId$ and from equation (i)

$$a = \frac{dv}{dt} = \frac{2\lambda I}{Bd}\frac{dx}{dt} = \frac{2\lambda I v}{Bd} = \frac{2\lambda I^2}{(Bd)^2}(R + 2\lambda x)$$

So, $F = BId + \dfrac{2\lambda m I^2}{(Bd)^2}(R + 2\lambda x)$

(ii) As the work done by force F per sec.

$$\frac{dW}{dt} = P = Fv = \left[BId + \frac{2\lambda m I^2}{(Bd)^2}(R+2\lambda x)\right]\left[\frac{R+2\lambda x}{Bd}.I\right]$$

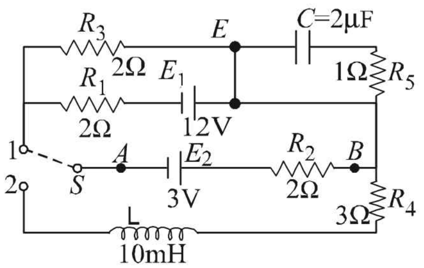
i.e., $P = \left[I^2(R+2\lambda x) + \dfrac{2\lambda m I^3}{B^3d^3}(R+2\lambda x)^2\right]$

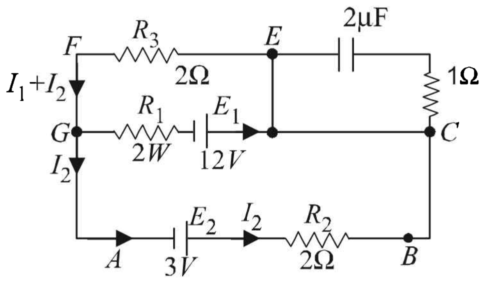
and heat produced per second, i.e., joule heat

$H = I^2(R + 2\lambda x)$

So, $f = \dfrac{H}{P} = \left[1 + \dfrac{2\lambda m I(R+2\lambda x)}{B^3d^3}\right]^{-1}$

7. **(a)** (i) In this case S and I are connected.





Using Kirchhoff's law in ABCDGA

$+3 - I_2 \times 2 - 12 + I_1 \times 2 = 0$

$2I_1 - 2I_2 = 9$ ...(i)

Applying Kirchhoff's law in DEFGD

$-2I_1 + 12 - (I_1 + I_2)2 = 0$

$\Rightarrow 2I_1 + I_2 = 6$ ...(ii)

From (i) and (ii) $I_1 = \frac{21}{6}$ amp.

$\therefore$ From (ii) $I_2 = -1$ amp.

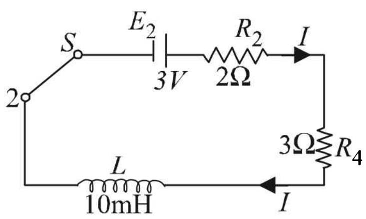
To find potential difference between A and B

$V_A + 3 - (-1) \times 2 = V_B \Rightarrow V_A - V_B = -5V$

The rate of production of heat in R_1

$$= I_1^2 R_1 = \left(\frac{21}{6}\right)^2 \times 2 = 24.5W$$

(b) (i) When the switch is put in position 2 then the active circuit will be as shown in the figure.



When the steady state current is reached then the inductor plays no role in the circuit

$E_2 = I(R_2 + R_4)$

$$\Rightarrow I = \frac{3}{5} = 0.6 \text{ amp.}$$

(ii) **KEY CONCEPT :** The growth of current in L–R circuit is given by the expression

$$I = I_0\left[1 - e^{-\frac{R}{L}t}\right]$$

When $I = \frac{I_0}{2}$, then $\frac{I_0}{2} = I_0\left[1 - e^{-\frac{R}{L}t}\right]$

$$\Rightarrow \frac{1}{2} = 1 - e^{\frac{-R}{L}t} \Rightarrow e^{-\frac{R}{L}t} = \frac{1}{2}$$

Taking log on both sides

$$\log_e e^{-\frac{R}{L}t} = \log_e \frac{1}{2}$$

$$\Rightarrow \frac{R}{L}t = 0.693 \Rightarrow t = 0.693\frac{L}{R} = \frac{0.6930 \times 10 \times 10^{-3}}{(2+3)}$$

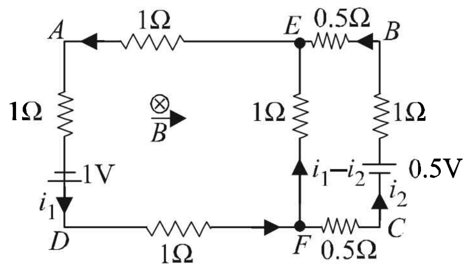
when $R = R_2 + R_4$

$\Rightarrow t = 1.386 \times 10^{-3}$ sec. Thus this much time is required for current to reach half of its steady value.

The energy stored by the inductor at that time is given

$$\text{by } E = \frac{1}{2}LI^2 = \frac{1}{2} \times 10 \times 10^{-3} \times \left(\frac{0.6}{2}\right)^2 = 4.5 \times 10^{-4}\,\text{J}$$

8. The equivalent circuit is drawn in the adjacent figure.



NOTE : As the magnetic field increases in the downward direction, an induced emf will be produced in the $AEFD$ as well as in the circuit $EBCF$ such that the current flowing in the loop creates magnetic lines of force in the upward direction (to the plane of paper).

Thus, the current should flow in the anticlockwise direction in both the loops.

Induced emf in loop $AEFD$

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}BA = -A\frac{dB}{dt} = -1 \times 1 = -1\text{volt}$$

Induced emf in loop $EBCF$

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}BA' = -A'\frac{dB}{dt} = -0.5 \times 1 = -0.5 \text{ volt}$$

Let the current flowing in the branch $EADF$ be i_1 and the current flowing in the branch $FCBE$ be i_2. Applying junction law at F, we get current in branch FE to be $(i_1 - i_2)$

Applying Kirchhoff's law in loop $EADFE$

$-1 \times i_1 - 1 \times i_1 + 1 - 1 \times i_1 - 1(i_1 - i_2) = 0$

$\Rightarrow 4i_1 - i_2 = 1$ ... (i)

Applying Kirchhoff's law in loop $EBCFE$

$+0.5i_1 - 0.5 + 1i_2 + 0.5i_2 - 1(i_1 - i_2) = 0$

$-i_1 + 3i_2 = 0.5$ ... (ii)

Solving (i) and (ii)

$11i_1 = 3.5$

$\Rightarrow \quad i_1 = 3.5/11 = \frac{7}{22} A$

Also $11 i_2 = 3$

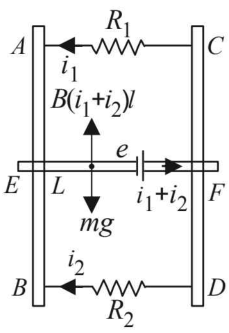
$\Rightarrow \quad i_2 = 3/11\, A = \frac{6}{22} A$

$\therefore$ Current in segment $AE = i_1 = \frac{7}{22} A$

Current in segment $BE = i_2 = \frac{6}{22} A$

Current in segment $EF = (i_1 - i_2) = \frac{1}{22} A$

9. **KEY CONCEPT :** We can understand the direction of flow of induced currents by imagining a fictitious battery to be attached between E and F. The direction of induced current can be found with the help of Lenz's law.



NOTE : P.d. across parallel combinations remains the same

Also, $\quad P_1 = e i_1 = 0.76$ W

and $\quad P_2 = e i_2 = 1.2$ W

$\therefore \quad \frac{i_1}{i_2} = \frac{1.76}{1.2} \Rightarrow i_1 = \frac{1.76}{1.2} i_2 \qquad \ldots$(ii)

The horizontal metallic bar L moves with a terminal velocity. This means that the net force on the bar is zero.

$\therefore \quad B(i_1 + i_2) = mg$

$\Rightarrow \quad i_1 + i_2 = \frac{mg}{B\ell} = \frac{0.2 \times 9.8}{0.6 \times 1} = \frac{49}{15}$ amp. ......(iii)

From (ii) and (iii)

$$\frac{1.76}{1.2} i_2 + i_2 = \frac{49}{15}$$

$\Rightarrow \quad i_2 = 2$ amp. $\Rightarrow \quad i_1 = \frac{19}{15}$ amp. $\Rightarrow \quad e = \frac{0.76}{19/15} = 0.6V$

The induced emf across L due to the movement of bar L in a magnetic field

$$e = Bv_T L \Rightarrow v_T = \frac{e}{BL} = \frac{0.6}{0.6 \times 1} = 1 \text{ m/s}$$

Also from (i),

$$R_1 = \frac{e}{i_1} = \frac{0.6}{19/15} = 0.47\Omega \text{ and } R_2 = \frac{e}{i_2} = \frac{0.6}{2} = 0.3\Omega$$

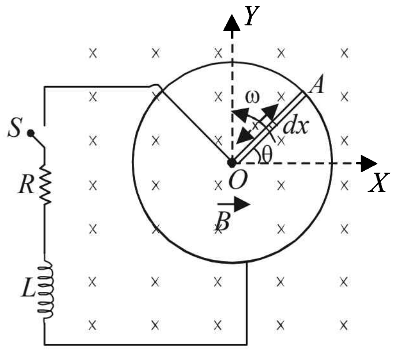
10. (i) (a) Let us consider a small length of metal rod dx at a distance x from the origin. Small amount of emf (de) induced in this small length (due to metallic rod cutting magnetic lines of force) is

$de = B(dx)v \qquad \ldots$(i)

where v is the velocity of small length dx

$v = x\omega \qquad \ldots$(ii)

$\therefore$ The total emf acoss the whole metallic rod OA is



$$e = \int_0^r Bx\omega\, dx = B\omega \left[\frac{x^2}{2}\right]_0^r = \frac{Br^2\omega}{2}$$

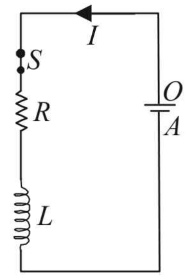
(b) The above diagram can be reconstructed as the adjacent figure. e is a constant. O will accumulate positive charge and A negative. When the switch S is closed, transient current at any time t, when current I is flowing in the circuit,

$I = I_0 (1 - e^{t/\tau})$

Here,

$I_0 = \frac{e}{R} = \frac{B\omega r^2}{2R}$

and $\tau = \frac{L}{R}$



Therefore, $I = \frac{B\omega r^2}{2R}[1 - e^{-\left(\frac{R}{L}\right)t}]$

(ii) In steady state,

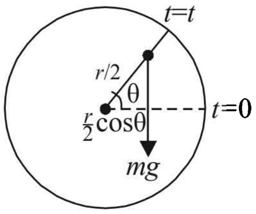
$I = \frac{B\omega r^2}{2R} \qquad [\because t$ has a large value and $e^{-\left(\frac{R}{L}\right)t} \to 0]$

NOTE : When current flows in the circuit in steady state, there is a power loss through the resistor.

Also since the rod is rotating in a vertical plane, work needs to be done to keep it at constant angular speed.

Power loss due to current I will be

$$P = I^2 R = \left(\frac{Br^2\omega}{2R}\right)^2 R$$



If torque required for this power is τ_1 then

$P = \tau_1 \omega$

$\Rightarrow \quad \tau_1 = \frac{B^2 r^4 \omega}{4R}$

Torque required to move the rod in circular motion against gravitational field

$$\tau_2 = mg \times \frac{r}{2}\cos\theta$$

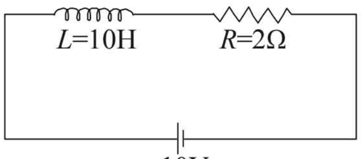
The total torque

$\tau = \tau_1 + \tau_2 \qquad$ (Clockwise)

$$\tau = \frac{B^2 r^4 \omega}{4R} + \frac{mgr}{2}\cos \omega t$$

The required torque will be of same magnitude and in anticlockwise direction. The second term will change signs as the value of cos θ can be positive as well as negative.

11. **KEY CONCEPT :** Let I_0 be the current at steady state. The magnetic energy stored in the inductor at this state will be



$$E = \frac{1}{2} L I_0^2 \qquad \ldots \text{(i)}$$

This is the maximum energy stored in the inductor. The current in the circuit for one fourth of this energy can be found as

$$\frac{1}{4} \times E = \frac{1}{2} L I^2 \qquad \ldots \text{(ii)}$$

Dividing equation (i) and (ii)

$$\frac{E}{E/4} = \frac{\frac{1}{2} L I_0^2}{\frac{1}{2} L I^2} \Rightarrow I = \frac{I_0}{2}$$

Also, $V = I_0 R$

$$\Rightarrow I_0 = \frac{V}{R} = \frac{10}{2} = 5 \text{ amp.} \quad \therefore I = \frac{I_0}{2} = \frac{5}{2} = 2.5 \text{ amp.}$$

The equation for growth of current in L-R circuit is

$$I = I_0 [1 - e^{-Rt/L}]$$

$$\Rightarrow 2.5 = 5[1 - e^{-2t/10}] \quad \Rightarrow \frac{1}{2} = 1 - e^{-t/5}$$

$$\Rightarrow t = 5 \log_e 2 = 2 \times 2.303 \times 0.3010 = 3.466 \text{ sec.}$$

12. **KEY CONCEPT :** If v is the velocity of the rod at any time t, induced emf is BvL and so induced current in the rod

$$I = \frac{\text{Induced e.m.f.}}{R} = \frac{BvL}{R}$$

Due to this current, the rod in the field B will experience a force

$$F = BIL = \frac{B^2 L^2 v}{R} \text{ (opposite to its motion)} \ldots (1)$$

So, equation of motion of the rod will be,

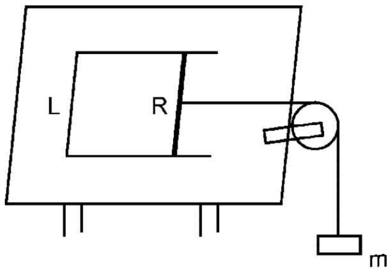
$T - F = 0 \times a$, i.e., $T = F$ [as rod is massless]

$$mg - T = ma \Rightarrow a = g - \frac{T}{m} = g - \frac{B^2 L^2 v}{mR} \qquad \ldots (2)$$

So rod will acquire terminal velocity when its acceleration is zero i.e.,

$$g - \frac{B^2 L^2 v_T}{mR} = 0 \text{ i.e. } v_T = \frac{mgR}{B^2 L^2};$$

For the case when velocity is $\frac{v_T}{2}$



$$v = \frac{v_T}{2} = \frac{mgR}{2B^2 L^2}$$

Substituting this value of velocity in eq. (2) we get

$$a = g - \frac{B^2 L^2}{mR} \times \frac{1}{2} \frac{mgR}{B^2 L^2} = g - \frac{1}{2} g = \frac{g}{2}$$

13. Suppose at $t = 0, y = 0$ and $t = t, y = y$

(a) Total magnetic flux = $\vec{B} . \vec{A}$

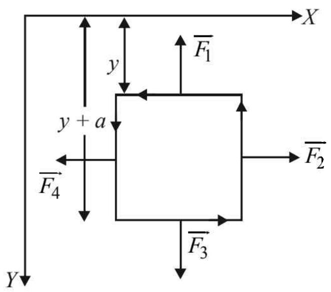
where $\vec{A} = a^2 \hat{k}$ and $\vec{B} = \frac{B_0 y}{a} \hat{k}$

$$\therefore \phi = \frac{B_0 y}{a} . a^2 = B_0 y a$$

Net emf., $e = -\frac{d\phi}{dt} = -B_0 a \frac{dy}{dt} = -B_0 a v(t)$

As total resistance = R

$$\therefore |i| = \frac{|e|}{R} = \frac{B_0 a v(t)}{R}$$



NOTE : Now as loop goes down, magnetic flux linked with it increases, hence induced current flows in such a direction so as to reduce the magnetic flux linked with it. Hence, induced current flows in anticlockwise direction.

(b) Each side of the cube will experience a force as shown (since a current carrying segment in a magnetic field experiences a force).

$$\vec{F_1} = i(\vec{\ell} \times \vec{B}) = i\left(-a\hat{i} \times \frac{B_0 y}{a} \hat{k}\right) = B_0 y (\hat{i} \times \hat{j});$$

$$\vec{F_3} = i\left(+a\hat{i} \times \frac{B_0 (y + a)}{a} \hat{k}\right) = i B_0 (y + a) \hat{j}$$

NOTE : $\vec{F}_2 = -\vec{F}_4$ and hence will cancel out each other.

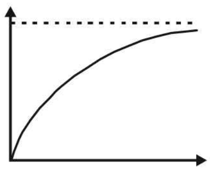
Net force, $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = -iB_0\, a\hat{j} = -\dfrac{B_0^2 a^2 v(t)}{R}\hat{j}$

(c) Total net force $= mg\,\hat{j} + \vec{F} = \left[mg - \dfrac{B_0^2 a^2 v(t)}{R}\right]\hat{j}$;

$$\therefore \quad m\frac{dv}{dt} = mg - \frac{B_0^2 a^2 v(t)}{R}$$

Integrating it, we get, $\displaystyle\int_0^v \frac{dv}{g - \frac{B_0^2 a^2 v(t)}{mR}} = \int_0^t dt$

$$\frac{\log\left[g - \frac{B_0^2 a^2 v(t)}{mR}\right]_0^{(v)t}}{\frac{-B_0^2 a^2}{mR}} = t$$



or $\quad \log\left[\dfrac{g - \frac{B_0^2 a^2 v(t)}{mR}}{g}\right] = -\dfrac{B_0^2 a^2 t}{mR}$

or $\quad 1 - \dfrac{B_0^2 a^2 v(t)}{mgR} = e^{-\left(B_0^2 a^2 t\right)/mR}$

or $\quad 1 - e^{-\left(B_0^2 a^2 t\right)/mR} = \dfrac{B_0^2 a^2}{mgR} v(t)$;

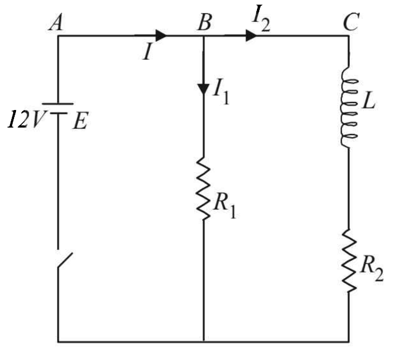
$\therefore \quad v(t) = \dfrac{mgR}{B_0^2 a^2}\left[1 - e^{-\left(B_0^2 a^2 t\right)/mR}\right]$

When terminal velocity is attained, $v(t)$ does not depend on t

$\therefore \quad v(t) = \dfrac{mgR}{B_0^2 a^2}$

14. This is a question on growth and rise of current.
GROWTH OF CURRENT : Let at any instant of time t the current be as shown in the figure.
Applying Kirchoff's law in the loop ABCDFGA we get, starting from G moving clockwise

$$E - L\frac{dI_2}{dt} - I_2 R_2 = 0$$



or $I_2 = \dfrac{E}{R_2}\left[1 - e^{-\frac{R}{L}t}\right]$

Also we know that the emf (V) produced across the inductor

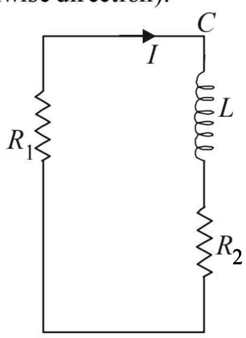
$$V = -\frac{d\phi}{dt} = -\frac{d}{dt}[LI_2] = -L\frac{dI_2}{dt}$$

$$= -L\frac{d}{dt}\left[\frac{E}{R_2}\left(1 - e^{\frac{-R_2}{L}t}\right)\right]$$

$V = -E\, e^{-\frac{R_2}{L}t}$. Here the negative sign shows the opposition to the growth of current.

$\therefore \quad V = 12\, e^{-\frac{2}{400\times 10^{-3}}t} = 12\, e^{-5t}$ volt

DECAY OF CURRENT : When the switch is opened, the branch AG is out of the circuit. Therefore, the current decays through the circuit $CBFDC$ (in clockwise direction).
Applying Kirchhoff's law



$$I(R_1 + R_2) - \left(-\frac{L\,dI}{dt}\right) = 0$$

$$\therefore \quad \frac{dI}{I} = -\left(\frac{R_1 + R_2}{L}\right)dt$$

$\therefore$ On integrating,

$$\int_{I_0}^{I} \frac{dI}{I} = -\frac{(R_1 + R_2)}{L}\int_0^t dt$$

$$\therefore \quad I = I_0\, e^{-\frac{(R_1+R_2)t}{L}}$$

Here, $\dfrac{R_1 + R_2}{L} = \dfrac{2+2}{400\times 10^{-3}} = 10$

and $I_0 = \dfrac{E}{R_1 + R_2} = \dfrac{12}{4} = 3$ A

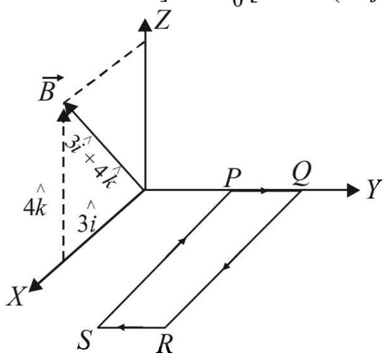
$\therefore \quad I = 3e^{-10t} A$, clockwise.

Alternatively, you may directly find the time constant $\tau = \dfrac{L}{R_1 + R_2}$ and use the equation $i = i_0 e^{-t/\tau}$ where $i_0 = 6A$

15. Let us consider the current in the clockwise direction in loop $PQRS$. Force on wire QR,

$$\vec{F}_{QR} = I(\vec{\ell} \times \vec{B}) = I[(a\hat{i}) \times (3\hat{i} + 4\hat{k})B_0]$$

$$= I B_0[3a\hat{i} \times \hat{i} + 4a\hat{i} \times \hat{k}] = IB_0[0 + 4a(-\hat{j})] = -4aB_0 I\hat{j}$$



Force on wire PS

$\vec{F}_{PS} = I(\vec{\ell} \times \vec{B}) = I[a(-\hat{i}) \times (3\hat{i} + 4\hat{k}) B_0] = 4aB_0 I\hat{j}$

Thus we see that force on QR is equal and opposite to that on PS and balance each other.

The force on RS is

$$\vec{F}_{RS} = I(\vec{\ell} \times \vec{B}) = I[b(-\hat{j}) \times (3\hat{i} + 4\hat{k}) B_0]$$

$$= I bB_0 [3\hat{k} - 4\hat{i}] \quad \text{... (i)}$$

The torque about PQ by this force is

$$\vec{\tau}_{RS} = \vec{r} \times \vec{F} = (\hat{i}\, a) \times (3\hat{k} - 4\hat{i}) IbB_0$$

$$= I\, abB_0 (3\hat{j}) \quad \text{... (ii)}$$

The torque about PQ due to weight of the wire $PQRS$ is

$$\tau = mg\left(\frac{a}{2}\right) \quad \text{... (iii)}$$

For the wire loop to be horizontal, we have to equate (ii) and (iii) $3IabB_0 = mg\dfrac{a}{2}$

$$\Rightarrow I = \frac{mg}{6bB_0} \quad \text{... (iv)}$$

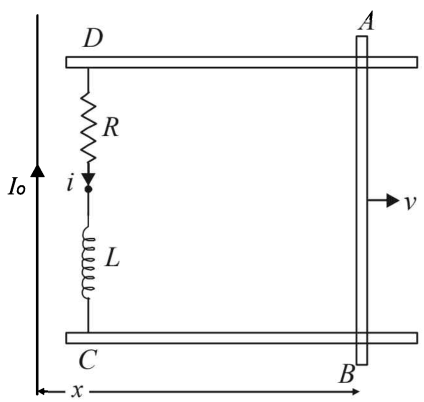
Therefore,

(a) The direction of current assumed is right. This is because torque due to mg and current are in opposite directions. Therefore, current is from P to Q.

(b) From (i), $\vec{F}_{RS} = IbB_0(3\hat{k} - 4\hat{i})$

(c) From (iv), $I = \dfrac{mg}{6aB_0}$

16. **(a)** **KEY CONCEPT :** As the metal bar AB moves towards the right, the magnetic flux in the loop $ABCD$ increases in the downward direction. By Lenz's law, to oppose this, current will flow in anticlockwise direction as shown in figure.



Applying Kirchhoff's loop law in $ABCD$, we get

$$\frac{d\phi}{dt} = iR + L\frac{di}{dt} \quad \text{... (i)}$$

(b) Let AB be at a distance x from the long straight wire at any instant of time t during its motion. The magnetic field at that instant at AB due to long straight current carrying wire is

$$B = \frac{\mu_0 I_0}{2\pi x}$$

The change in flux through $ABCD$ in time dt is

$$d\phi = B\,(dA) = B\,\ell\, d\,x$$

Therefore, the total flux change when metal bar moves from a distance x_0 to $2x_0$ is

$$\Delta\phi = \int_{x_0}^{2x_0} B\ell dx = \ell\int_{x_0}^{2x_0} \frac{\mu_0 I_0}{2\pi x} dx = \frac{\mu_0 I_0 \ell}{2\pi}[\log_e x]_{x_0}^{2x_0}$$

$$= \frac{\mu_0 I_0 \ell}{2\pi}\log_e 2 \quad \text{... (ii)}$$

The charge flowing through resistance R in time T is

$$q = \int_0^T i dt = \int_0^T \frac{1}{R}\left[E_{\text{induced}} - L\frac{di}{dt}\right]dt \quad \text{[from eq. (i)]}$$

$$= \frac{1}{R}\int_0^T E_{\text{induced}} dt - \frac{L}{R}\int_0^{i_1} d\,i = \frac{1}{R}(\Delta\,\phi) - \frac{L}{R}i_1$$

$$q = \frac{1}{R}\left[\frac{\mu_0 I_0\,\ell}{2\pi}\log_e 2\right] - \frac{L}{R}i_1 \quad \text{from eq. (ii)}$$

(c) When the metal bar AB is stopped, the rate of change of magnetic flux through $ABCD$ becomes zero.

From (i),

$$iR = -L\frac{di}{dt}$$

$$\int_T^{2T} dt = \frac{L}{R}\int_{i_1}^{i_1/4} \frac{di}{i}$$

$$T = -\frac{L}{R}\log_e \frac{i_1/4}{i_1} \Rightarrow \frac{L}{R} = \frac{T}{2\log_e 2}$$

17. **(a)** Let us consider a small strip of thickness dx as shown in the figure.

The magnetic field at this strip

$$B = B_A + B_B$$

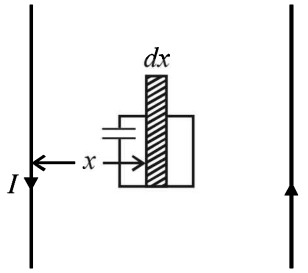
(Perpendicular to the plane of paper directed upwards)

$$= \frac{\mu_0}{2\pi}\frac{I}{x} + \frac{\mu_0}{2\pi}\frac{I}{(3a - x)}$$

B_A = Magnetic field due to current in wire A

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} + \frac{1}{3a - x}\right]$$

B_B = Magnetic field due to current in wire B



Small amount of magnetic flux passing through the strip of thickness dx is

$$d\phi = B \times adx = \frac{\mu_0 Ia \times 3a\,dx}{2\pi x(3a-x)}$$

Total flux through the square loop

$$\phi = \int_a^{2a} \frac{\mu_0 I \times 3a^2}{2\pi} \frac{dx}{x(3a-x)} = \frac{\mu_0 Ia}{\pi} \ln 2$$

$$= \frac{\mu_0 a \ln(2)}{\pi}(I_0 \sin \omega t)$$

The emf produced

$$e = \left| -\frac{d\phi}{dt} \right| = \frac{\mu_0 a I_0 \omega}{\pi} \ln(2) \cos \omega t$$

Charge stored in the capacitor

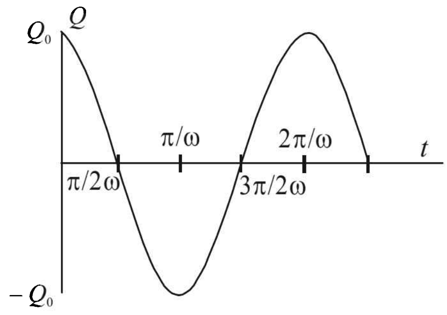
$$q = C \times e = C \times \frac{\mu_0 a I_0 \omega}{\pi} \ln(2) \cos \omega t \quad ...(i)$$

∴ Current in the loop

$$i = \frac{dq}{dt} = \frac{C \times \mu_0 a I_0 \omega^2}{\pi} \ln(2) \sin \omega t$$

$$\therefore \quad i_{max} = \frac{\mu_0 a I_0 \omega^2 C \ln(2)}{\pi}$$

(b) From (i), the graph between charge and time is



Here, $q_0 = \dfrac{C \times \mu_0 a I_0 \omega \ln(2)}{\pi}$

18. Given, $V_{rms} = 220\,V$

$\nu = 50$ Hz, $L = 35$ mH, $R = 11\Omega$

Impedance

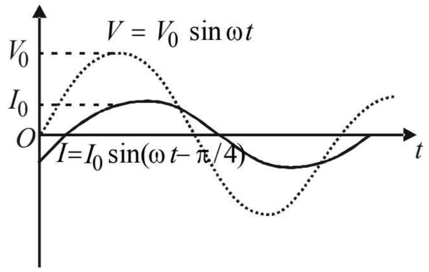
$$Z = \sqrt{(\omega L)^2 + R^2} = 11\sqrt{2}\,\Omega$$

also, $I_0 = \dfrac{V_0}{Z}$

$V_0 = V_{rms}\sqrt{2}$ $\quad \therefore I_0 = \dfrac{V_{rms}\sqrt{2}}{Z} = 20A$

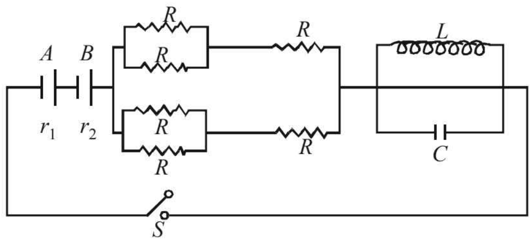
$\cos\phi = \dfrac{R}{Z} = \dfrac{1}{\sqrt{2}}$ $\quad \therefore \phi = \dfrac{\pi}{4}$

∴ graph is given by.



19. **NOTE :** After a long time capacitor will be fully charged, hence no current will flow through capacitor and all the current will flow from inductor. Since current is D.C., resistance of L is zero.

$$\therefore \quad R_{eq} = \left(\frac{R}{2} + R\right) \times \frac{1}{2} + r_1 + r_2 = \frac{3R}{4} + r_1 + r_2$$



$$I = \frac{\varepsilon + \varepsilon}{R_{eq}} \Rightarrow I = \frac{2\varepsilon}{R_{eq}} = \frac{2\varepsilon}{(3R/4) + r_1 + r_2}$$

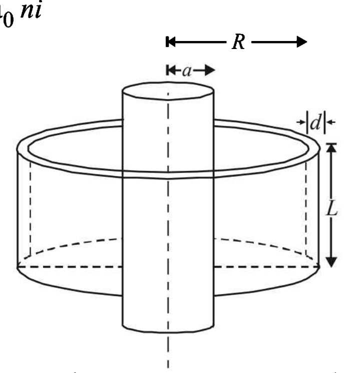
Potential drop across A is

$$\varepsilon - Ir_1 = 0 \quad \Rightarrow \quad \varepsilon = \frac{2\varepsilon}{(3R/4) + r_1 + r_2} r_1$$

$$\Rightarrow \quad r_1 = r_2 + 3R/4 \quad \text{or} \quad R = \frac{4}{3}(r_1 - r_2)$$

20. **KEY CONCEPT :** The magnetic field in the solenoid is given by

$B = \mu_0 ni$



$\Rightarrow \quad B = \mu_0 n i_0 \sin \omega t \qquad [\because i = i_0 = \sin \omega t \text{ (given)}]$

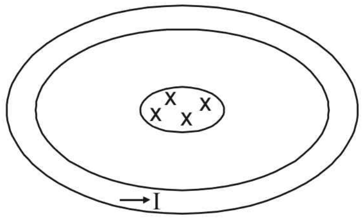
The magnetic flux linked with the solenoid

$$\phi = \vec{B}.\vec{A} = BA\cos 90° = (\mu_0 n i_0 \sin\omega t)(\pi a^2)$$

∴ The rate of change of magnetic flux through the solenoid

$$\frac{d\phi}{dt} = \pi\mu_0 n a^2 i_0\, \omega\cos\omega t$$

The same rate of change of flux is linked with the cylindrical shell. By the principle of electromagnetic induction, the induced emf produced in the cylindrical shell is



TOP VIEW

$$e = -\frac{d\phi}{dt} = -\pi\mu_0 n a^2 i_0\, \omega\cos\omega t \quad ...(i)$$

The resistance offered by the cylindrical shell to the flow of induced current I will be

$$R = \rho\frac{\ell}{A}$$

Here, $\ell = 2\pi R$ and $A = L\times d$

$$\therefore \quad R = \rho\frac{2\pi R}{Ld} \quad ...(ii)$$

The induced current I will be

$$I = \frac{|e|}{R} = \frac{[\pi\mu_0 n a^2 i_0\, \omega\cos\omega t]\times Ld}{\rho\times 2\pi R}$$

$$\Rightarrow \quad I = \frac{\mu_0 n a^2 L d\, i_0\, \omega\cos\omega t}{2\rho R}$$

F. Match the Following

1. **A-r,s,t; B-q,r,s,t; C-p,q; D-q,r,s,t**

The following are the important concepts which are applied in the given situation.

(i) For DC circuit, in steady state, the current I through the capacitor is zero. In case of L-C circuit, the potential difference across the inductor is zero and that across the capacitor is equal to the applied potential difference. In case of L-R circuit, the potential difference across inductor is zero across resistor is equal to the applied voltage.

(ii) For AC circuit in steady state, I_{rms} current flows through the capacitor, inductor and resistor. The potential difference across resistor, inductor and capacitor is proportional to I.

(iii) For DC circuit, for changing current, the potential difference across inductor, capacitor or resistor is proportional to the current.

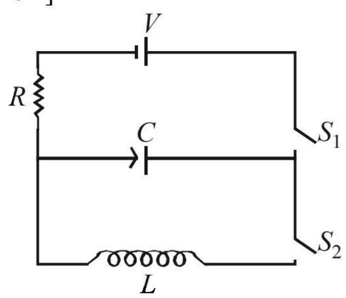
G. Comprehension Based Questions

1. **(b)** For charging of $R - C$ circuit, $Q = Q_0[1 - e^{-t/\tau}]$ when the charging is complete, the potential difference between the capacitor plates will be V. The charge stored in this case will be maximum.

Therefore, $Q_0 = CV$.

When $t = 2\tau$, $Q = CV\left[1 = e^{\frac{-2\tau}{\tau}}\right]$

$= CV[1 - e^{-2}]$

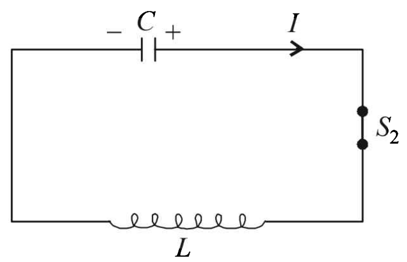


2. **(d)** The instantaneous charge on plates at any time t during discharging is

$$Q = Q_0\cos\omega t$$

∴ Instantaneous current,

$$I = \frac{dQ}{dt} = Q_0\,\omega\sin\omega t$$



∴ The magnitude of maximum current

$I_{max} = Q_0\omega$

Here $Q_0 = CV$ and $\omega = \frac{1}{\sqrt{LC}}$

$$\therefore \quad I_{max} = CV\times\frac{1}{\sqrt{LC}} = V\sqrt{\frac{C}{L}}$$

3. **(c)** Apply Kirchhoff's law in the circuit

$$\frac{Q}{C} - L\frac{dI}{dt} = 0 \Rightarrow \frac{Q}{C} = L\frac{dI}{dt}$$

$$\Rightarrow \quad Q = LC\frac{d}{dt}\left(-\frac{dQ}{dt}\right) = -LC\frac{d^2Q}{dt^2}$$

4. **(a)** For step up transformer $\frac{N_s}{N_p} = \frac{V_s}{V_p} \Rightarrow \frac{10}{1} = \frac{V_s}{4000}$

∴ $V_s = 40{,}000$ V

For step down transformer $\frac{N_p}{N_s} = \frac{V_p}{V_s}$

$= \frac{40,000}{200} = \frac{200}{1}$

(a) is the correct option.

5. **(b)** We know that P = V × I

$\therefore \quad I = \frac{P}{V} = \frac{600 \times 1000}{4000}$

$\therefore \quad I = 150\,A$

Total resistance = 0.4 × 20 = 8 Ω

∴ Power dissipated as heat = $I^2R = (150)^2 \times 8$ = 180,000W = 180 kW

$\therefore \quad \%\ \text{loss} = \frac{180}{600} \times 100 = 30\%$

(b) is the correct option.

6. **(b)** $\int \vec{E}.\vec{dl} = \frac{-d\phi}{dt} = -\frac{d}{dt}(B\pi R^2) = -\pi R^2 \frac{dB}{dt}$

$= -\pi R^2 B$

$\therefore E \times 2\pi R = -\pi R^2 B$

$\therefore E = \frac{-BR}{2}$

(b) is the correct option.

7. **(b)** Given $M = \gamma L$

$\therefore \quad M = \gamma m\omega R^2$

$\therefore \quad M = \gamma m(\Delta\omega)R^2$ ...(1)

But $\Delta\omega = \frac{Q \times B}{2m}$ ...(2)

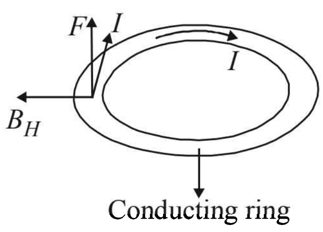
From (1) and (2) $\Delta M = -\gamma m \left(\frac{QB}{2m}\right) R^2 = \frac{-\gamma BQR^2}{2}$

The negative sign shows that change is opposite to the direction of B.

(b) is the correct option.

H. Assertion & Reason Type Questions

1. **(a)** As shown in the figure the horizontal component of the magnetic field interacts with the induced current produced in the conducting ring which produces an average force in the upward direction. (Fleming's left hand rule).



I. Integer Value Correct Type

1. **(4)** Time constant = RC

Impedance $= \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

Given impedance = $R\sqrt{1.25}$

$\therefore \quad R\sqrt{1.25} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

$\therefore RC = \frac{2}{\omega} = \frac{2}{500} \times 1000$ ms

$\therefore RC = 4$ ms

2. **(7)** The magnetic field due to current carrying wire at the location of square loop is

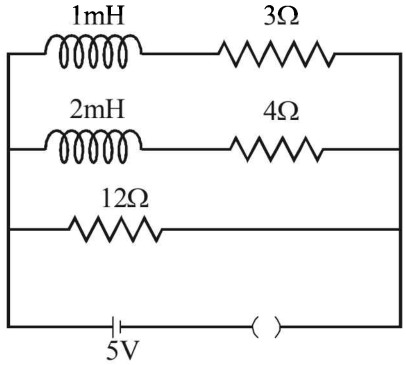
$$B = \frac{\mu_0}{4\pi}\frac{2\pi i R^2}{(R^2 + 3R^2)^{3/2}} = \frac{\mu_0 i}{16R}$$

The mutual induction

$$M = \frac{N\phi}{i} = \frac{2}{i}\left[\frac{\mu_0 i}{16R} \times a^2 \cos 45°\right]$$

$$\therefore \quad M = \frac{\mu_0 a^2}{2^{7/2} R}$$

3. **(8)**



At t = 0 $I_{min} = \frac{5}{12}$

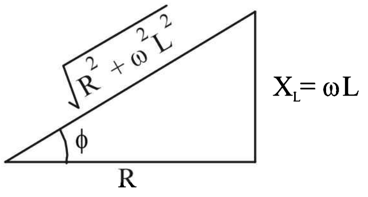
At t = ∞ $I_{max} = \frac{5}{R_{eq}} = \frac{5}{3/2} = \frac{10}{3}$

$\left[\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{4} + \frac{1}{12} = \frac{8}{12}\right]$

$\therefore \frac{I_{max}}{I_{min}} = \frac{10}{3} \times \frac{12}{5} = 8$

Section-B JEE Main/ AIEEE

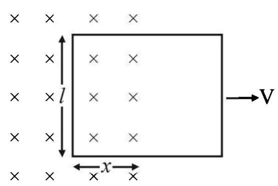
1. **(b)** The impedance triangle for resistance (R) and inductor (L) connected in series is shown in the figure.



Power factor $\cos\phi = \dfrac{R}{\sqrt{R^2+\omega^2L^2}}$

2. **(d)** The induced emf is

$$e = \frac{-d\phi}{dt} = -\frac{d(\vec{B}.\vec{A})}{dt} = \frac{-d(BA\cos 0°)}{dt}$$



$$\therefore e = -B\frac{dA}{dt} = -B\frac{d(\ell\times x)}{dt} = -B\ell\frac{dx}{dt} = -B\ell v$$

3. **(d)** These three inductors are connected in parallel. The equivalent inductance L_p is given by

$$\frac{1}{L_p} = \frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = \frac{3}{3} = 1$$

$\therefore L_p = 1$

4. **(b)** $N_p = 140, N_s = 280, I_p = 4A, I_s = ?$

For a transformer $\dfrac{I_s}{I_p} = \dfrac{N_p}{N_s}$

$$\Rightarrow \frac{I_s}{4} = \frac{140}{280} \Rightarrow I_s = 2\,A$$

5. **(b)** Mutual conductance depends on the relative position and orientation of the two coils.

6. **(d)** $e = -\dfrac{\Delta\phi}{\Delta t} = \dfrac{-\Delta(LI)}{\Delta t} = -L\dfrac{\Delta I}{\Delta t}$

$$\therefore |e| = L\frac{\Delta I}{\Delta t} \Rightarrow 8 = L\times\frac{4}{0.05}$$

$$\Rightarrow L = \frac{8\times 0.05}{4} = 0.1\text{H}$$

7. **(c)** When the capacitor is completely charged, the total energy in the L.C circuit is with the capacitor and that energy is

$$E = \frac{1}{2}\frac{Q^2}{C}$$

When half energy is with the capacitor in the form of electric field between the plates of the capacitor we get

$\dfrac{E}{2} = \dfrac{1}{2}\dfrac{Q'^2}{C}$ where Q' is the charge on one plate of the capacitor

$$\therefore \frac{1}{2}\times\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q'^2}{C} \Rightarrow Q' = \frac{Q}{\sqrt{2}}$$

8. **(a)** Laminated core provide less area of cross-section for the current to flow. Because of this, resistance of the core increases and current decreases thereby decreasing the eddy current losses.

9. **(a)** D.C. ammeter measure average current in AC current, average current is zero for complete cycle. Hence reading will be zero.

10. **(d)** Since the phase difference between L & C is π,
$\therefore$ net voltage difference across $LC = 50 - 50 = 0$

11. **(b)** $\dfrac{\Delta\phi}{\Delta t} = \dfrac{(W_2 - W_1)}{t}$

$$R_{tot} = (R + 4R)\Omega = 5R\Omega$$

$$i = \frac{nd\phi}{R_{tot}dt} = \frac{-n(W_2 - W_1)}{5Rt}$$

($\because W_2$ & W_1 are magnetic flux)

12. **(b)** $\phi = \vec{B}.\vec{A}$; $\phi = BA\cos\omega t$

$$\varepsilon = -\frac{d\phi}{dt} = \omega BA\sin\omega t\ ;\ i = \frac{\omega BA}{R}\sin\omega t$$

$$P_{inst} = i^2R = \left(\frac{\omega BA}{R}\right)^2 \times R\sin^2\omega t$$

$$P_{avg} = \frac{\int_0^T P_{inst}\times dt}{\int_0^T dt} = \frac{(\omega BA)^2}{R}\frac{\int_0^T \sin^2\omega t\,dt}{\int_0^T dt} = \frac{1}{2}\frac{(\omega BA)^2}{R}$$

$$\therefore P_{avg} = \frac{(\omega B\pi r^2)^2}{8R} \qquad \left[A = \frac{\pi r^2}{2}\right]$$

13. **(a)** For resonant frequency to remain same LC should be const. LC = const

$$\Rightarrow LC = L'\times 2C \Rightarrow L' = \frac{L}{2}$$

14. **(b)** $\ell = 1$m, $\omega = 5$ rad/s, $B = 0.2\times 10^{-4}\,T$

$$\varepsilon = \frac{B\omega\ell^2}{2} = \frac{0.2\times 10^{-4}\times 5\times 1}{2} = 50\mu V$$

15. (c) Relative velocity $= v + v = 2v$

$\therefore$ emf. $= B.l(2v)$

16. (d) For maximum power, $X_L = X_C$, which yields

$$C = \frac{1}{(2\pi n)^2 L} = \frac{1}{4\pi^2 \times 50 \times 50 \times 10}$$

$\therefore C = 0.1 \times 10^{-5} F = 1\mu F$

17. (a) Phase difference for R–L circuit lies between $\left(0, \frac{\pi}{2}\right)$

18. (b) Power factor $= \cos\phi = \frac{R}{Z} = \frac{12}{15} = \frac{4}{5} = 0.8$

19. (a) **KEY CONCEPT :** The charging of inductance given by, $i = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$

$$\frac{i_0}{2} = i_0(1 - e^{-\frac{Rt}{L}}) \Rightarrow e^{-\frac{Rt}{L}} = \frac{1}{2}$$

Taking log on both the sides,

$$-\frac{Rt}{L} = \log 1 - \log 2$$

$$\Rightarrow t = \frac{L}{R}\log 2 = \frac{300 \times 10^{-3}}{2} \times 0.69$$

$\Rightarrow t = 0.1$ sec.

20. (b) Mutual inductance $= \frac{\phi}{I} = \frac{BA}{I}$

$$[\text{Henry}] = \frac{[MT^{-1}Q^{-1}L^2]}{[QT^{-1}]} = ML^2Q^{-2}$$

21. (c) Across resistor, $I = \frac{V}{R} = \frac{100}{1000} = 0.1\ A$

At resonance,

$$X_L = X_C = \frac{1}{\omega C} = \frac{1}{200 \times 2 \times 10^{-6}} = 2500$$

Voltage across L is

$I\,X_L = 0.1 \times 2500 = 250\text{V}$

22. (d) $e = -\frac{d\phi}{dt} = -\frac{d(N\vec{B}.\vec{A})}{dt}$

$$= -N\frac{d}{dt}(BA\cos\omega t) = NBA\omega\sin\omega t$$

$\Rightarrow e_{max} = NBA\omega$

23. (b) $\phi = 10t^2 - 50t + 250$

$$e = -\frac{d\phi}{dt} = -(20t - 50)$$

$e_{t=3} = -10\ V$

24. (a) Initially, when steady state is achieved,

$$i = \frac{E}{R}$$

Let E is short circuited at $t = 0$. Then

At $t = 0$, $i_0 = \frac{E}{R}$

Let during decay of current at any time the current flowing is $-L\frac{di}{dt} - iR = 0$

$$\Rightarrow \frac{di}{i} = -\frac{R}{L}dt \Rightarrow \int_{i_0}^{i}\frac{di}{i} = \int_0^t -\frac{R}{L}dt$$

$$\Rightarrow \log_e \frac{i}{i_0} = -\frac{R}{L}t \Rightarrow i = i_0\, e^{-\frac{R}{L}t}$$

$$\Rightarrow i = \frac{E}{R}e^{-\frac{R}{L}t} = \frac{100}{100}e^{\frac{-100\times10^{-3}}{100\times10^{-3}}} = \frac{1}{e}$$

25. (c) **KEY CONCEPT :** We know that power consumed in a.c. circuit is given by, $P = E_{rms}.I_{rms}\cos\phi$

Here, $E = E_0 \sin\omega t$

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

which implies that the phase difference, $\phi = \frac{\pi}{2}$

$$\therefore P = E_{rms}.I_{rms}.\cos\frac{\pi}{2} = 0 \qquad \left(\because \cos\frac{\pi}{2} = 0\right)$$

26. (a) **KEY CONCEPT :** $I = I_o\left(1 - e^{-\frac{R}{L}t}\right)$

(When current is in growth in LR circuit)

$$= \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{5}{5}\left(1 - e^{-\frac{5}{10}\times 2}\right) = (1 - e^{-1})$$

27. (d) $M = \frac{\mu_0 N_1 N_2 A}{\ell} = \frac{4\pi \times 10^{-7} \times 300 \times 400 \times 100 \times 10^{-4}}{0.2}$

$= 2.4\pi \times 10^{-4}\text{H}$

28. (c) Growth in current in LR_2 branch when switch is closed is given by

$$i = \frac{E}{R_2}[1 - e^{-R_2 t/L}] \Rightarrow \frac{di}{dt} = \frac{E}{R_2}.\frac{R_2}{L}.e^{-R_2 t/L} = \frac{E}{L}e^{-\frac{R_2 t}{L}}$$

Hence, potential drop across

$$\text{L} = \left(\frac{E}{L}e^{-R_2 t/L}\right)L = Ee^{-R_2 t/L} = 12e^{-\frac{2t}{400\times10^{-3}}}$$

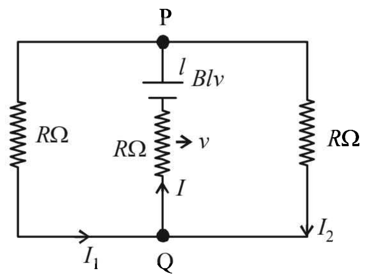
$= 12e^{-5t}\text{V}$

29. (b) Due to the movement of resistor R, an emf equal to Blv will be induced in it as shown in figure clearly,

$I = I_1 + I_2$

Also, $I_1 = I_2$

Solving the circuit, we get



$I_1 = I_2 = \frac{Blv}{3R}$

and $I = 2I_1 = \frac{2Blv}{3R}$

30. (c) At $t = 0$, no current will flow through L and R_1

$\therefore$ Current through battery $= \frac{V}{R_2}$

At $t = \infty$,

effective resistance, $R_{eff} = \frac{R_1 R_2}{R_1 + R_2}$

$\therefore$ Current through battery $= \frac{V}{R_{eff}} = \frac{V(R_1 + R_2)}{R_1 R_2}$

31. (d) When capacitance is taken out, the circuit is LR.

$\therefore \tan\phi = \frac{\omega L}{R}$

$\Rightarrow \omega L = R\tan\phi = 200 \times \frac{1}{\sqrt{3}} = \frac{200}{\sqrt{3}}$

Again, when inductor is taken out, the circuit is CR.

$\therefore \tan\phi = \frac{1}{\omega CR}$

$\Rightarrow \frac{1}{\omega c} = R\tan\phi = 200 \times \frac{1}{\sqrt{3}} = \frac{200}{\sqrt{3}}$

Now, $Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$

$= \sqrt{(200)^2 + \left(\frac{200}{\sqrt{3}} - \frac{200}{\sqrt{3}}\right)^2} = 200\,\Omega$

Power dissipated $= V_{rms} I_{rms} \cos\phi$

$= V_{rms} \cdot \frac{V_{rms}}{Z} \cdot \frac{R}{Z} \left(\because \cos\phi = \frac{R}{Z}\right)$

$= \frac{V^2_{rms} R}{Z^2} = \frac{(220)^2 \times 200}{(200)^2} = \frac{220 \times 220}{200} = 242$ W

32. (c) Induced emf $= vB_H l = 1.5 \times 5 \times 10^{-5} \times 2 = 15 \times 10^{-5}$
$= 0.15$ mV

33. (a) Energy stored in magnetic field $= \frac{1}{2}$ Li^2

Energy stored in electric field $= \frac{1}{2}\frac{q^2}{C}$

$\therefore \frac{1}{2}Li^2 = \frac{1}{2}\frac{q^2}{C}$

Also $q = q_0 \cos\omega t$ and $\omega = \frac{1}{\sqrt{LC}}$

On solving $t = \frac{\pi}{4}\sqrt{LC}$

34. (b) We have, $V = V_0(1 - e^{-t/RC})$

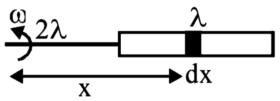
$\Rightarrow 120 = 200\left(1 - e^{-t/RC}\right)$

$\Rightarrow t = RC$ in (2.5)

$\Rightarrow R = 2.71 \times 10^6\,\Omega$

35. (d) Because of the Lenz's law of conservation of energy.

36. (d) Here, induced e.m.f.



$e = \int_{2\ell}^{3\ell} (\omega x) B dx = B\omega \frac{[(3\ell)^2 - (2\ell)^2]}{2} = \frac{5B\ell^2\omega}{2}$

37. (a) As we know, Magnetic flux, $\phi = B.A$

$\frac{\mu_0 (2)(20 \times 10^{-2})^2}{2[(0.2)^2 + (0.15)^2]} \times \pi(0.3 \times 10^{-2})^2$

On solving

$= 9.216 \times 10^{-11} = 9.2 \times 10^{-11}$ weber

38. (c) Charge on he capacitor at any time t is given by

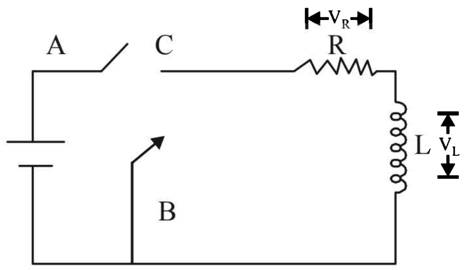
$q = CV(1 - e^{t/\tau})$

at $t = 2\tau$

$q = CV(1 - e^{-2})$

39. (c) Applying Kirchhoff's law of voltage in closed loop

$-V_R - V_C = 0 \Rightarrow \frac{V_R}{V_C} = -1$



40. (b) $I(0) = \frac{15 \times 100}{0.15 \times 10^3} = 0.1$A

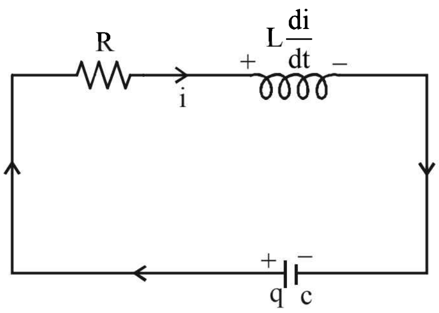
$I(\infty) = 0$

$I(t) = [I(0) - I(\infty)]\, e^{\frac{-t}{L/R}} + i(\infty)$

$I(t) = 0.1\, e^{\frac{-t}{L/R}} = 0.1\, e^{\frac{R}{L}}$

$I(t) = 0.1\, e^{\frac{0.15 \times 1000}{0.03}} = 0.67$mA

41. **(c)** From KVL at any time t



$$\frac{q}{c} - iR - L\frac{di}{dt} = 0$$

$$i = -\frac{dq}{dt} \Rightarrow \frac{q}{c} + \frac{dq}{dt}R + \frac{Ld^2q}{dt^2} = 0$$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{Lc} = 0$$

From damped harmonic oscillator, the amplitude is given by $A = A_o e - \frac{dt}{2m}$

Double differential equation $\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$

$$Q_{max} = Q_o e - \frac{Rt}{2L} \Rightarrow Q^2_{max} = Q^2_o e - \frac{Rt}{L}$$

Hence damping will be faster for lesser self inductance.

42. **(c)** $\vec{F}_1 = \vec{F}_2 = 0$

because of action and reaction pair

43. **(b)** Here

$$i = \frac{e}{\sqrt{R^2 + X_L^2}} = \frac{e}{\sqrt{R^2 + \omega^2 L^2}} = \frac{e}{\sqrt{R^2 + 4\pi^2 v^2 L^2}}$$

$$10 = \frac{220}{\sqrt{64 + 4\pi^2 (50)^2 L}} \quad [\because R = \frac{V}{I} = \frac{80}{10} = 8]$$

On solving we get

$L = 0.065\,H$